

Using High, Low, and Closing Data to Estimate Covariance

J.G. Wade and C.L. Zirbel
Bowling Green State University

Efficient estimation of volatility and covariance of securities is problem of tremendous practical importance in finance and risk management. Most commonly, this estimation is performed on the basis of period data such as daily or weekly closing prices. However, for many instruments such as stock and bonds, and other quantities of interest such as exchange rates, historical data is widely available not only for daily closing values but also for intraday extremal (high and low) data.

In 1984, C. A. Ball and W. N. Torous derived the joint distribution for the periodic high, low, and ending value for a driftless geometric Brownian motion, with volatility as a parameter. On the basis of this they demonstrated a highly efficient maximum likelihood estimator for volatility. In 2002, P. M. Lindholdt extended this work to the case of non-zero drift.

In the present paper, we extend these ideas to the problem of estimating covariance of two geometric Brownian motions, on the basis of the high, low and closing values for each. We derive the joint PDF, and present some preliminary numerical results demonstrating the use of maximum likelihood estimation for covariance with this PDF.