Advanced Topics in General Insurance

Exam GIADV

Date: Friday, May 3, 2024

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 13 questions numbered 1 through 13 with a total of 60 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 and σ^2 can be typed as sigma^2.

   b) Calculations should be done in Excel and entered as formulas. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit. Rows can be inserted to the answer input area as required to provide space for your answer.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

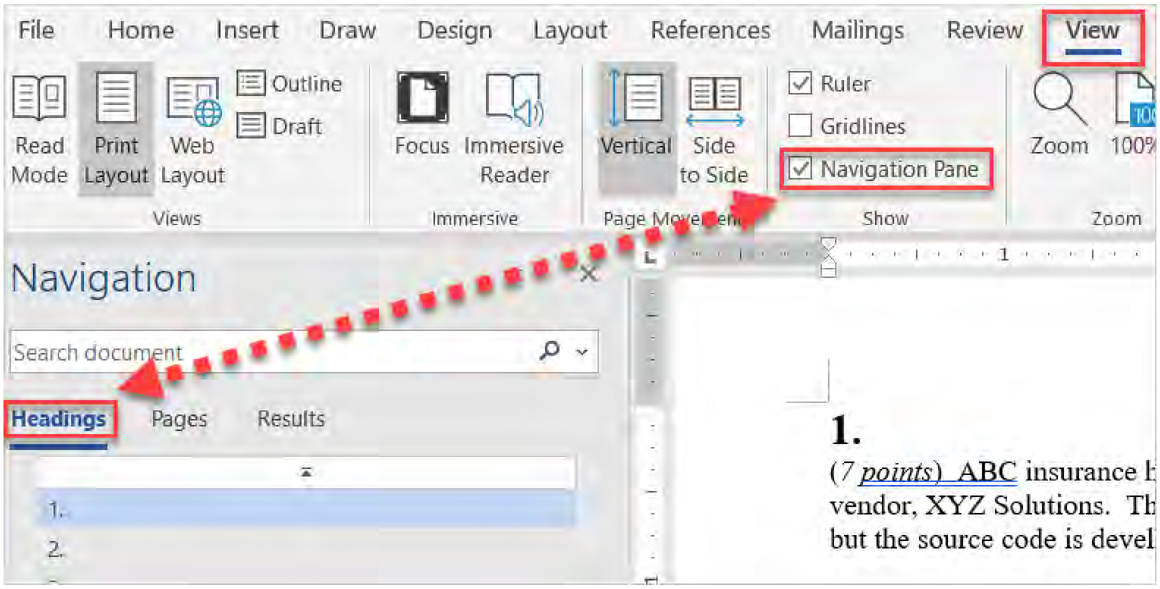
3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before the five-minute upload period expires.

Navigation Instructions

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



1.

Provide the response for this question in the Excel spreadsheet.

(*5 points*) A reinsurer is pricing a proportional treaty and is considering various adjustable features to manage profitability.

You are given the following information:

- Aggregate loss distribution:

Loss in Millions	Cumulative Probability
1	0.0526
2	0.2119
3	0.4058
4	0.5815
5	0.7191
6	0.8182
7	0.8855
8	0.9295
9	0.9573
10	0.9746
11	0.9850
12	0.9913
13	0.9949
14	0.9971
15	0.9984
16	0.9991
17	0.9995
18	0.9997
19	0.9998
20	0.9999

- Ceding commission is 30%
- Brokerage fees are 5%
- Other Expenses are 2%
- Treaty Premium is 8,000,000

(a) (*1.5 points*) Calculate the probability of a combined ratio of more than 100%.

1. **Continued**

You are considering using a loss corridor with the following feature:

> Ceding company will reassume 75% of the losses in the loss ratio layer from 60% to 100%.

(b) (*1 point*) Calculate the expected loss ratio after the loss corridor.

You are further considering a sliding scale commission, to apply to loss ratios calculated **after** the loss corridor. The sliding scale commission has the following features:

- Provisional commission is 30%
- Minimum commission is 15% at a 70% loss ratio
- Commission sliding 1:1 to 25% at a 60% loss ratio
- Commission sliding 0.5:1 to a maximum of 35% at a 40% loss ratio

(c) (*1.5 points*) Calculate the expected combined ratio.

(d) (*1 point*) Assess whether the sliding scale commission is balanced.

2.
(*4 points*)

(a) (*1 point*) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding individual risk rating.

(i) The schedule rating adjustment is typically applied _______ premium discounts. [*after, before*]

(ii) An experience modification factor of _______ is referred to as a credit modification. [*greater than 0, greater than 1, less than 0, less than 1*]

(iii) Increasing the cap applied to claims _______ the responsiveness of an experience rating formula. [*decreases, does not affect, increases*]

(iv) D-ratio curves relate to _______ in experience rating. [*application of premium discounts, determination of credibility, limiting of claims*]

(b) (*2 points*) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding insurer dividend plans.

(i) Insurers offer dividend plans to U.S. insureds for _______ coverage. [*commercial automobile, professional liability, workers compensation*]

(ii) Dividend plans closely resemble _______ rating plans. [*prospective, retrospective, schedule*]

(iii) Dividend plans are also referred to as _______. [*participating policies, predictive rating plans, risk-control plans*]

(iv) Dividend payments may require approval by the _______. [*insurer's board of directors, insurer's shareholders, regulatory authority*]

(v) In a sliding-scale dividend plan, the insured's claims experience _______ dividend payments. [*affects, does not affect*]

(vi) An insurer's board of directors may _______ dividend payments for all dividend plan policies. [*not reject, reject*]

(vii) Dividend payments occur after _______. [*the end of the policy period, the filing of the financial statements, settlement of the claims on the policy*]

(viii) A combined dividend plan is a combination of the sliding-scale and _______ dividend plans. [*flat, schedule-rated, split-rated*]

2. Continued

(c) (*1 point*) Describe the use of safety groups for U.S. workers compensation dividend plans.

ANSWER:

3.

(*5 points*) You are reviewing your assistant's report on reserves (the full report is not provided here). Your assistant has applied Clark's stochastic reserving model using the Cape Cod method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-x/\theta}$ where x is in months.

Your assistant provided an Excel worksheet with all their calculations. You will find them on the "Q03" tab in the supplied Excel file.

You are to critique the extracts below from the report.

- In some cases, the report extracts and supporting calculations may be accurate and all you need to do is note that.
- However, if there are errors, you are to identify them and provide an explanation of why the report extract or calculation is incorrect.
- If the error is wording or an explanation, provide the correct wording along with an explanation of why the report is incorrect.
- If the error is in a calculation, you need only identify the error. You do not need to modify the spreadsheet or calculate the correct number(s).
- Assume that your assistant successfully maximized their version of the likelihood function.
- Assume any formulas that apply to multiple cells are either correct for all such cells or have the same error in all such cells.

Provide your response in this document, and not the Excel spreadsheet.

(a) (*1 point*) Critique Part 1.

Part 1: *I have been provided with a loss triangle representing accident years 2016-2023. All possible twelve-month development periods are represented. I noticed that one of the increments is negative. While the algorithm provides results for this case, Clark's procedure should not be used when there are negative increments.*

ANSWER:

3. **Continued**

(b) (*1 point*) Critique Part 2.

Part 2: *I have performed maximum likelihood estimation of the two parameters. The calculations are in cells G7:K43 and the maximizing values are in cells H44 and H45. The values that maximize the likelihood function are* $\theta = 37.44$ *and ELR = 0.5506.*

ANSWER:

(c) (*1 point*) Critique Part 3.

Part 3: *I have calculated the estimate of the scale factor in cells M7:M44 as* $\hat{\sigma}^2 = 309.1$.

ANSWER:

(d) (*1 point*) Critique Part 4.

Part 4: *I have calculated the loss reserve estimate for all years in cell F60 as 31,103. I have further calculated the process standard deviation of the loss reserve in cell F62 as 3,101. Assuming a normal distribution, two standard deviations provide 95% confidence. Therefore, we can be 95% confident that the actual development will be between 24,901 and 37,304.*

ANSWER:

(e) (*1 point*) Critique Part 5.

Part 5: *Clark recommends several graphs that can be used to verify if the assumptions of the model hold. Two such graphs are on the worksheet. One graph plots normalized residuals against increment age and the other plots them against the expected increments. The second graph shows a curved, rather than horizontal pattern. Hence the underlying assumptions may not hold.*

ANSWER:

4.

Provide the response for this question in the Excel spreadsheet.

(*8 points*) You are interested in determining the variability of unpaid claim estimates. The triangle of cumulative paid claims data is presented below. It is assumed that all claims are fully developed after ten years.

Accident Year (AY)	**Development Year (DY)**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	30,374	76,584	104,035	124,753	139,829	148,994	156,583	164,187	165,169	169,267
2	30,346	83,030	114,913	133,802	146,265	157,740	163,287	167,517	170,685	
3	35,782	93,320	131,071	145,988	160,291	171,415	179,806	185,071		
4	32,230	87,349	124,761	148,440	163,804	175,738	184,092			
5	41,583	109,196	147,524	176,075	187,492	204,788				
6	70,281	183,436	234,094	273,204	291,046					
7	71,668	177,489	229,819	258,084						
8	59,878	150,125	191,380							
9	61,721	146,504								
10	47,807									

(a) (*2 points*) Calculate the following amounts using the chain ladder (CL) method:

(i) Age-to-age factors

(ii) Expected values for each DY (starting with DY 2), based on the previous DY, for which there are observed values

Mack suggests two plots that can be used to check the validity of the assumptions underlying the CL estimates.

(b) (*2 points*) Construct the following scatterplots for each of DYs 1 to 3:

(i) Development along with a fitted regression line

(ii) Weighted residuals versus values from the previous DY

(c) (*1 point*) Determine the validity of the assumptions underlying the CL estimates based upon part (b). Justify your determination.

Assume that the scatterplots indicate that the assumptions are not valid. Mack provides two other formulas for calculating age-to-age factors that could be tested.

(d) (*0.5 points*) State one of these alternatives. Do not do any calculations.

4. Continued

Venter proposes an alternative test that involves age-to-age factors. His test is to perform a regression on the increments versus the prior cumulative value.

(e) (*0.5 points*) Input the appropriate values to perform this regression analysis.

(f) (*1 point*) Determine the validity of the assumptions underlying the CL estimates based upon the results of the regression analysis in part (e). Justify your determination.

(g) (*1 point*) Propose an alternative model that is more consistent with the results of the regression analysis in part (e).

5.

Provide the response for this question in the Excel spreadsheet.

(*4 points*) A reinsurance company is renewing three accounts, X, Y and Z, each of which is exposed to three possible claim events, 1, 2 and 3, which are independent of each other.

Event (*i*)		**Loss for Account**		
i	**p(*i*)**	**X**	**Y**	**Z**
1	0.5%	40,000	20,000	10,000
2	1.1%	20,000	10,000	5,000
3	2.1%	5,000	1,000	500

- p(*i*) represents the probability of event *i*.
- The risk load multiplier, λ, equals 0.000024.

(a) (*3.5 points*) Calculate the renewal risk load for each account using the following methods:

(i) Marginal Variance

(ii) Shapley

(b) (*0.5 points*) Demonstrate that the Shapley method is renewal additive.

6.

(*5 points*) You are given the following information regarding an insurance policy:

- The number of losses incurred by a risk in a policy period is a random variable, N.
- The loss payment amount is a random variable X.
- $F(x)$ is the cumulative distribution function of X.
- $G(x) = 1 - F(x)$
- $E\{X\}$ is the expected value of X.
- $E\{N\}$ is the expected value of N.
- L is the policy limit.
- b is the basic limit.

Integrations using $F(x)$ and $G(x)$ may be included in your answers as follows:

- $\mathrm{A}[v, w] = \int_v^w x\,dF(x)$
- $\mathrm{B}[v, w] = \int_v^w (x - v)\,dF(x)$
- $\mathrm{C}[v, w] = \int_v^w G(x)\,dx$

(a) (*3 points*) Show the formula for each of the following amounts from this policy:

(i) Expected payment per loss using the layer method

(ii) Expected payment per loss using the size method

(iii) Expected total payments in a policy period

(iv) Increased limits factor (ILF) for limit k with basic limit b

(v) Derivative of the ILF for limit k with basic limit b

(vi) Expected basic limits loss after constant inflation of factor a (i.e., $x \to x'$ in which $x' = ax$)

ANSWER:
(i)
(ii)
(iii)
(iv)
(v)
(vi)

6. Continued

Now assume that the policy coverage is only for losses exceeding an amount R subject to a limit L. The payment may be expressed as the function $h(X;\ R,\ L)$ in which:

$$h(X;R,L)=\begin{cases}0, & 0<X\le R\\ X-R, & R<X\le(R+L)\\ L, & (R+L)<X\end{cases}$$

(b) (*1.5 points*) Show the formula for each of the following amounts from this policy:

(i) Expected payment per loss using the layer method

(ii) Expected payment per loss using the size method

ANSWER:
(i)
(ii)

(c) (*0.5 points*) State what is required of ILFs to pass the consistency test.

ANSWER:

7.

Provide the response for this question in the Excel spreadsheet.

(*4 points*) You are estimating the premium asset for retrospectively rated polices using the PDLD method developed by Teng and Perkins. You are given the following information for a retrospectively rated book of business.

Basic premium (BP)	31.5 million
Standard premium (SP)	140 million
Expected loss ratio (ELR)	87.5%
Loss conversion factor (LCF)	1.19
Tax multiplier (TM)	1.02

Retrospective Adjustment	**Expected percentage of loss emerged (EPLE)**	**Loss elimination ratio from per accident limit (LEPA)**	**Loss elimination ratio from retro formula max and min (LEMM)**
First	82.30%	2.1%	3.0%
Second	91.10%	2.6%	4.6%
Third	97.80%	3.0%	7.5%
Fourth	100.00%	3.2%	9.0%

(a) (*2.5 points*) Calculate the cumulative premium development to loss development (CPDLD) ratio for each retrospective adjustment period using the formula approach.

(b) (*0.5 points*) State the formula to estimate the premium asset that includes the CPLD ratio as one of the elements in the formula.

(c) (*0.5 points*) Identify two situations where an empirical approach to estimating PDLD ratios would be preferred to the formula approach.

(d) (*0.5 points*) Provide a reason the PDLD method might be preferred to Fitzgibbon’s method.

8.

Provide the response for this question in the Excel spreadsheet.

(*4 points*) You are calculating a risk margin for an insurance company with two lines of business using the method set out in "A Framework for Assessing Risk Margins."

Line of Business	Central Estimate (in millions)	
	Outstanding claims (OSC)	**Premium liabilities (PL)**
Auto	24	50
Home	16	30

Line of Business	Coefficient of Variation (CoV)			
	Internal Systemic Risk		**External Systemic Risk**	
	OSC	**PL**	**OSC**	**PL**
Auto	4.0%	6.0%	8.0%	5.0%
Home	10.0%	12.5%	6.0%	10.0%

	Internal systemic risk correlation matrix			
	Auto OSC	**Auto PL**	**Home OSC**	**Home PL**
Auto OSC	100%	50%	25%	0%
Auto PL	50%	100%	25%	0%
Home OSC	25%	25%	100%	0%
Home PL	0%	0%	0%	100%

- The total independent risk CoV for the company's insurance liabilities is 4.5%.
- For external systemic risk, assume there is no correlation between lines of business and between the components of insurance liabilities (OSC and PL).
- The three sources of uncertainty (independent risk, internal systemic risk, and external systemic risk) are assumed to be mutually independent.
- Assume that the underlying distribution of insurance liabilities for this company is the Normal distribution.
- The z-value of the 80th percentile of the Normal distribution is 0.8416.

(a) (*1 point*) Describe two considerations why correlation effects exist within internal systemic risk.

8. **Continued**

(b) (*3 points*) Calculate the following for the company:

(i) Total internal systemic risk CoV

(ii) Total external systemic risk CoV

(iii) Total consolidated CoV for all sources of risk

(iv) Risk margin at the 80% adequacy level

9.

Provide the response for this question in the Excel spreadsheet.

(*4 points*)

(a) (*0.5 points*) Describe two examples of contracts where the risk transfer is "reasonably self-evident."

The metrics expected reinsurer deficit (ERD) and risk coverage ratio (RCR) have been presented as being superior to other metrics such as value-at-risk (VaR) or tail value-at-risk (TVaR) for gauging risk transfer.

(b) (*1 point*) Describe two advantages of using ERD and RCR versus using VaR and TVaR.

Another metric used in the testing of risk transfer is right-tailed deviation (RTD). A risk transfer test called maximum qualified premium (Max QP) has been proposed in which the test uses a multiple, α, of RTD.

You are to use the Max QP test to determine whether risk transfer exists in a reinsurance contract given the following information:

- A catastrophe excess of loss reinsurance contract for the layer 300 million excess of 400 million.
- The reinsurance premium is a fixed amount of 48 million payable to the reinsurer at inception of the contract.

Reinsured losses (millions)	Probability
0	94.0%
50	3.3%
100	1.6%
200	0.8%
300	0.3%

(c) (*2.5 points*) Determine whether risk transfer exists in this contract using the Max QP test with α equal to 4.

10.

Provide the response for this question in the Excel spreadsheet.

(*4 points*) You are performing a reserve analysis for the layer of 150,000 excess of 50,000 as of December 31, 2023.

Accident Year	Reported Claims (000) at 50,000 Limits					
	12	**24**	**36**	**48**	**60**	**72**
2018	3,742	4,715	5,116	5,372	5,533	5,588
2019	2,986	3,703	4,036	4,258	4,343	
2020	3,846	4,884	5,289	5,606		
2021	3,571	4,492	4,851			
2022	3,214	4,050				
2023	3,055					

Accident Year	Reported Claims (000) at 200,000 Limits					
	12	**24**	**36**	**48**	**60**	**72**
2018	N/A	N/A	N/A	N/A	N/A	7,706
2019	N/A	N/A	N/A	N/A	7,435	
2020	N/A	N/A	N/A	7,692		
2021	N/A	N/A	7,047			
2022	N/A	5,630				
2023	4,239					

	50,000 Limits	**200,000 Limits**
Calendar Year 2023 Earned Premium (000)	6,000	11,500
Expected Loss Ratio	70%	65%

- The 200,000 limits loss data for prior reporting periods is not available due to a database issue.
- There is no loss development beyond 72 months.
- All the claims have been trended to December 31, 2023.
- Total ultimate claims for 200,000 limits for all accident years are 46,986,000.
- There were no rate changes from 2021 through 2023.
- The annual premium trend is 3%.

(a) (*1 point*) Calculate the total IBNR for the layer of 150,000 excess of 50,000 using volume-weighted average loss development factors.

10. **Continued**

(b) (*0.5 points*) Explain why the expected method would be a viable alternative to the development method.

(c) (*0.5 points*) Describe a limitation of using the expected method.

(d) (*1 point*) Calculate the AY 2023 IBNR for the layer using the expected method.

You are considering the increased limit factor (ILF) method to estimate the layer reserves. The ILF applicable to premium for 200,000 limits relative to 50,000 limits at the December 31, 2023 cost level is 1.9. You use the ultimate claims for the 50,000 limits from part (a).

(e) (*1 point*) Calculate the AY 2023 IBNR for the layer using the ILF method.

11.

(*4 points*) During the 1970s, circumstances in the United States created a shift to claims-made coverage for certain types of insurance.

(a) (*1 point*) Describe these circumstances.

ANSWER:

(b) (*0.5 points*) Identify two reasons that this shift to claims-made coverage was not as prevalent outside of the United States.

ANSWER:

A key concept of claims-made coverage is the retroactive date.

(c) (*0.5 points*) Define the claims-made coverage retroactive date.

ANSWER:

Marker and Mohl identified five principles of claims-made ratemaking.

(d) (*2 points*) State four of these principles.

ANSWER:

12.

Provide the response for this question in the Excel spreadsheet.

(*5 points*)

(a) (*1.5 points*) Describe three ways that a self-insured retention (SIR) differs from the use of a deductible.

The elimination ratio approach is one of the ways to determine deductible factors.

(b) (*1.5 points*) Provide the following with respect to an insurer's application of this approach.

(i) Definition of elimination ratio

(ii) Formula for elimination ratio

The reduction in premium is not proportional to the size of the deductible for many lines of general insurance, particularly automobile physical damage coverages and personal property insurance.

(c) (*0.5 points*) Explain why this should be expected.

You are given the following:

- An insured property is valued at 200,000.
- The amount insured is 100,000.
- The insurance policy includes a coinsurance percentage of 60%.
- A loss occurred and the loss amount is 40,000.

(d) (*1.5 points*) Determine the amount the insurer would pay to the insured for this loss under the following scenarios. State any assumptions required.

(i) The policy has no deductible.

(ii) The policy has a deductible of 2,500.

13.

Provide the response for this question in the Excel spreadsheet.

(*4 points*) A reinsurance broker has proposed that ABC Reinsurance Company (ABC Re) provide a finite risk cover without reinstatements to JKL Insurance Company (JKL) with the following terms:

- Annual Premium: 20,000,000
- Occurrence Limit: 150,000,000
- Profit Commission: 80% after 10% margin on Annual Premium
- Additional Premium: 50% of (Loss + Margin – Annual Premium)

(a) (*0.5 points*) Calculate the nominal rate on line.

(b) (*0.5 points*) Calculate the underwriting loss (excluding expenses) to ABC Re if a loss fully exhausts the limit.

(c) (*0.5 points*) Calculate the premium for an equivalent traditional risk cover.

(d) (*0.5 points*) Calculate the rate on line for an equivalent traditional risk cover.

A catastrophe model indicates that a loss will fully exhaust the limit once every 10 years, and that the probability of a partial loss is negligible.

ABC Re has concluded that this proposal is not acceptable.

(e) (*2 points*) Construct a counterproposal that should be acceptable to both ABC Re and JKL. Justify your answer.

****END OF EXAMINATION****