2017 Enterprise Risk Management Symposium

April 20–21, 2017, New Orleans

ERM Stochastic Analysis Tools: Risk Drivers Revealed, Part II: Conditional Conditional Tail Expectation

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Abstract

Most stochastic Enterprise Risk Managment (ERM) models for life insurance examine only the resultant output (specifically the economic capital), and thereby separate the model results from the key input model assumptions, such as the term structure of interest rates. With ERM modeling, the calculation of economic capital (EC) is very expensive due to the complexity of the products and regulatory controls placed on the industry along with the requirement of a large number of scenarios to produce the empirical distribution of EC. Certain techniques have arisen to reduce this modeling cost, such as grid computing and replicating portfolios. Even with these reductions, a high cost is exacted from the enterprise. However, despite all of the resources dedicated to the generation of EC, the analysis of results is frequently limited to the determination of the empirical distribution and an obligatory examination of the relationships of the five worst and five best scenarios to the EC.

In 2012, the use of quantile regression (QR) was introduced to the modeling of the conditional VaR. In this paper, conditional Conditional Tail Expectation (CTE) regression is introduced to develop understanding of how risk drivers affect the average capital or reserves beyond a conditional VaR threshold. This simple technique provides additional tools for EC and reserve dashboards, especially as principle-based approaches (PBA) continue to expand within the insurance industry.

*Though this appears to be a typographical error or form of ‘double-speak’, there is a valid use of the term ‘conditional conditional‘ that will be made clear within the paper.

The above is accomplished by applying least squares regression on subsets through the use of residuals from the QR.

1 Introduction

In (Craighead, 2012) the use of quantile regression (QR) to capital Value at Risk (VaR) analysis is introduced. Quantile regression was originally developed by (Koenker and Basset, 1978). This paper will make use of it to:

- Gain further understanding around how risk drivers affect capital models where VaR is a metric
- Extract additional information from stochastic models of capital
- Aid in the development of risk dashboards that relates the VaR to changing economic conditions.

The application of a new form of regression allows the examination of capital models with CTE metrics in a similar fashion to how quantile regression can be used with VaR metrics. Just as QR addresses the above issues using VaR metrics in capital modeling, the new regression allows the use of CTE metrics.

With the tongue firmly planted in cheek, the term 'double speak' names this new regression as Conditional Conditional Tail Expectation(CCTE) and this naming convention will become apparent in the examination of the two separate conditional components around the regression.

CCTE regression is built on top of the VaR quantile regression through the use of QR residuals.

In the next Section 2, the introductory section of (Craighead, 2012) is expanded where model use and its limitations and benefits is discussed.

Section 3 briefly discusses the illustrative business model that makes use of the input scenarios and capital results to form the basis of the analysis. Many more details on the data and scenarios are in (Craighead, 2012).

In Section 4 a model is created that uses one risk driver and examines the of sensitivity of CCTE to that driver. In a subsection, the use of CCTE to derive additional understanding around the stochastic reserve component of the Principles Based Reserve methodology within the U.S. insurance industry is discussed.

In Section 5 the use of a CCTE regression model to construct an economic dashboard is demonstrated.

The strengths and weaknesses, a brief discussion of potential uses and future research of the CCTE method is discussed in Section 6.

Appendix A demonstrates both quantile and CCTE regression in R. Appendix B disclaims this work with any affiliation with specific organizations both past and present. Finally, the bibliography closes the paper.

2 Model Use - Strength and Weaknesses

In the life insurance industry, regulation and/or professional standards require a practitioner to conduct computer simulations on different lines of business to determine when the business performs poorly. Business is modeled as accurately as possible, allowing for interest and asset performance, changing premiums and expense loads. Assumptions on the claims count or amount distributions may or may not be set. In addition, many other assumptions are set, such as the term structure of interest rates, future interest rates, projected stock market returns, asset default probabilities, policyholder psychology, and the relationships of decrements to the level of interest rates or the stock market. Computer simulations reveal the behavior of the business relative to these assumptions. The actual statistical distribution of the business model results is unknown, however computer simulation results are assumed to be representative (within some degree of confidence) in certain areas of interest, such as the extreme tail. After model validation, within some degree of confidence, economic capital or stand-alone capital must be calculated. Also, there is a need to observe the potential risks associated with either the enterprise, product or line of business.

Computer simulations of complex corporate models become very expensive in processing time as the number of scenarios increases. The need to obtain a timely answer often outweighs the need for information from additional scenarios.

In ERM life insurance modeling this cost is reduced by using either predictive modeling, see (Craighead, 2008) or replicating portfolio approaches, see (Schrager, 2008) or (Burmeister et al, 2010).

Most computer business models are limited by the knowledge practioners have about the basic assumptions used. Care must be taken in the use of these models. At a fundamental level, the models are neither correct nor assumed to be accurate. However, the benefit of using the computer to model actual business products and lines is that an understanding of the different risks to that product or line are revealed. Once there is understanding, consideration can be taken to reduce the impact of any given risk. Such methods include product redesign, reserve strengthening, deferred expense write downs, asset hedging strategies, stopping rules (rules that recommend when to get out of a market), derivative positions and reinsurance, or overcapitalization.

Once basic understanding of the risks is gained, which leads to a design, say, a hedge strategy, one must remember that these models are not accurate, due to oversimplification of the model, lack of knowledge and insight, lack of confidence in the assumptions, or incorrect computer code. One cannot trust the model output as the "truth," but can trust the knowledge and insight that one gains from the process of modeling. If done correctly one knows both the strengths and weaknesses of the model. For instance, when constructing a hedge to protect against the risks demonstrated by the model, one must not implement a hedge that optimizes against areas of model weakness. Ultimately, the model does not tell one what to do, but the model does makes one more informed to make business decisions.

It is important to keep a clear perspective when using multiple economic scenarios in computer simulations. One can gain significant insight about the risk exposure from the economy using stochastic simulation. One realizes that only one path actually emerges as in the recent economic meltdown. Therefore, the practitioner must continually evaluate the economy and make reasoned business decisions to maintain existing business and to acquire new business.

The risk appetite of company management must also govern these business decisions. Insolvency must be considered and avoided. However, the practitioner cannot remove all risk of insolvency, because the cost of the associated hedges becomes so prohibitive that the company is unable to conduct business. Accordingly, the practitioner should understand where the product or business line places the company at risk and be able to communicate to upper management the specific risk exposure. For a further discussion of the balancing act between company profit and insolvency risk see (Craighead,2008).

ERM practitioners, valuation actuaries, asset/liability management actuaries, CFOs and CROs of insurance companies confront issues that are vast and complex, including:

- Calculating the probability and/or impact of bankruptcy either by scenario testing or by determining the company's value at risk.
- Determining the initial capital allocation for a new line of business.
- Assuring that reserves are adequate for new and existing lines of business.
- Understanding how different lines of business are sensitive to the level of interest rates, corporate spreads, volatility of other economic indicators (such as stock indices), and the changes in the levels of these variables.
- Estimating other risks to which the company is exposed in a timely fashion.
- Pricing complex policy features to obtain profitability, while maintaining a competitive market position.
- Aiding in the design and pricing of dynamic hedges to reduce the risk of extreme events.
- Designing and pricing the securitization of various cashflows to reduce risk based capital requirements and various types of reserves such as XXX or AXXX.
- Revising and designing investment strategies to improve the return on assets that back company liabilities.

All of the above issues require timely and accurate valuation of different complex corporate models. When conducting the analysis on models the practitioner goes through the following model life cycle:

- Collect relevant data.
- Make relevant assumptions.
- Construct the model.
- Validate the model for reasonableness.
- Apply the model to solve a problem or understand the impact of changing conditions.

- Revise the model.

After a corporate model is constructed the practitioner uses the results in several ways. Some of these are:

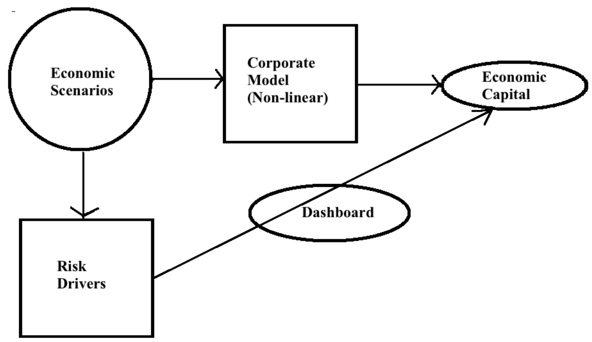
- Gain insight on the business modeled.
- Determine risks to the company.
- Observe the scenarios that give adverse model results.
- Increase reserves, create hedges or make product enhancements to reduce the risk exposure or adverse results.

The internal company standards and the external regulatory controls require the practitioner to determine risk levels from corporate models. It is of paramount importance to understand the impact that different economic drivers, product designs or investment/disinvestment strategies have on the behavior of a corporate model. This includes the determination of when (and how often) model results from scenarios fall in 'bad' locations. This knowledge allows one to interpret the potential magnitude of the company's risk exposure. While adverse results occur relatively infrequently in scenario testing (unless alternative volatility assumptions are considered), the practitioner desires to gain more knowledge of these adverse results without paying the cost of projecting additional scenarios to increase the number of "hits" in the region of adverse results needed for statistical validity.

These adverse locations are discovered by first placing a valuation of economic capital on the company's position, scenario by scenario. These valuations are then sorted and put in an increasing or decreasing order. From these ordered results, the location of the adverse results is found at either the highest or lowest valuations. The study and analysis of ordered or sorted samples is done using either order or extreme value statistics or the theory of records. Due to modeling cost, there is a need to approximate the relationship between the input economic scenarios and the EC output results without additional computer processing. Also, if one is able to target the location of adverse results when developing this relationship, all the better.

Through a model office or a corporate model and more-so the understanding arising from the use of those models strengthens decision making. Frequently, practitioners make reasoned decisions using a few deterministic

Figure 1: Concept of Risk Drivers



scenarios instead of a full suite of stochastic scenarios, however, though they may understand the underlying mechanics, they do not understand the likelihood of the impact of a risk unless a larger suite of scenarios are used. As more scenarios are used, the complexity increases and the loss of understanding of the mechanics increases and this leads to the proverbial situation of not being able to see the forest because of all of the trees in view. But ignorance that arises from complexity is not always a bad thing. It forces the modeler or the business professional to broaden their skill set to gain deeper insight, which leads to further product improvements or at least an understanding of model limitations.

With the advance of technology there are now new techniques from predictive analytics or data science that can be applied to these complex situations, and allow the practitioner to gain understanding of the model behavior between the scenario input and the corporate results.

The relationship between the scenario and the corporate results is outlined in Figure 1 where there is a non-linear computer corporate model that takes economic scenarios as input and produces certain model output, which represents the EC of the corporate model. Next, a risk driver is defined to be a function of the economic scenarios through time that distills the most informative characteristics of the economic scenarios, which have an impact on the model output. For example, the extraction of the time series of the 90-day Treasury bill rate from each scenario would be a potential risk driver. Another example is the time series of the spread of the 10-year Treasury note over the 90-day Treasury bill rate.

The dashboard model can be either a linear, a nonlinear, or even a predictive approximation of the EC at specific percentiles or averages of the EC above or below that specific percentile. This simpler model displays the relationship between the risk drivers and the results from original non-linear corporate model.

Next section looks at the data used to construct the examples.

3 Business Model-Input Economic Scenarios and Economic Capital

For illustrative purposes, 10,000 economic scenarios are used, which were generated from the process outlined in (Craighead, 2012). This process was one of the first used by Nationwide in the determination of reserve adequacy in the early 1990's. It is a real world process that has arbitrage within the yield curve and scenarios.

The projection horizon is 20 years with yield curves varying annually. The capital model output is the Equivalent Value of Accumulated Surplus (EVAS).[1] These EVAS values are obtained at the end of the projection period of 20 years and are discounted back to the valuation date. These are somewhat liberal in that if the company became insolvent in some year prior to year 20, but then recovers subsequently, there is no knowledge of that event contained in the corresponding twenty-year EVAS value.

The specific business model processed in 1993 is lost to history and the EVAS values have been modified to no longer resemble any of the original values from 1993. However, even though the scenario generation technique as well as the EVAS that were determined from these scenarios are dated, they still supply a rich enough environment to demonstrate the power of CCTE regression.

[1]Equivalent value of accumulated surplus is somewhat similar in concept to a present value, which is scenario dependent. It is also dependent upon the investment strategy used and is obtained by dividing the surplus at the end of the projection period by a growth factor. This factor represents the multiple by which a block of assets grows from the valuation date to the end of the period of interest. It is computed by accumulating existing assets or an initial lump-sum investment under the interest scenario in question on an after tax basis with the initial investment and any reinvestments being made using the selected investment strategy. The growth factor is the resulting asset amount at the end of the projection period divided by the initial amount at the valuation date, (Sedlak, 1997).

Figure 2: Associated Graphs of the Capital

(a) Density (b) S-Curve

Graphs of the density and S-curve of the capital are in Figure 2a and Figure 2b.

The basic statistics on the specific EVAS values are in Table 1.

Variable	**n**	**Min**	$\mathbf{q_1}$	$\widetilde{\mathbf{x}}$	$\bar{\mathbf{x}}$	$\mathbf{q_3}$	**Max**	**s**	**IQR**
LOB	10000	-16.1	19.8	25.1	23.1	28.3	34.9	7.4	8.6

Table 1: LOB Capital Statistics

Since only the interest rate scenarios are available for the data, the risk driver is restricted to be the change in the 10-year Treasury bond rates from the input scenarios. This is denoted by Y_t^{10} within the formulas, but $t1, t2, \ldots t20$ in the regressions.

In (Craighead, 2012) the spread between the 10-year Treasury bond rates and the 90-day Treasury bill rates was also considered, but this study is eliminated for space considerations.

4 Modeling of CCTE

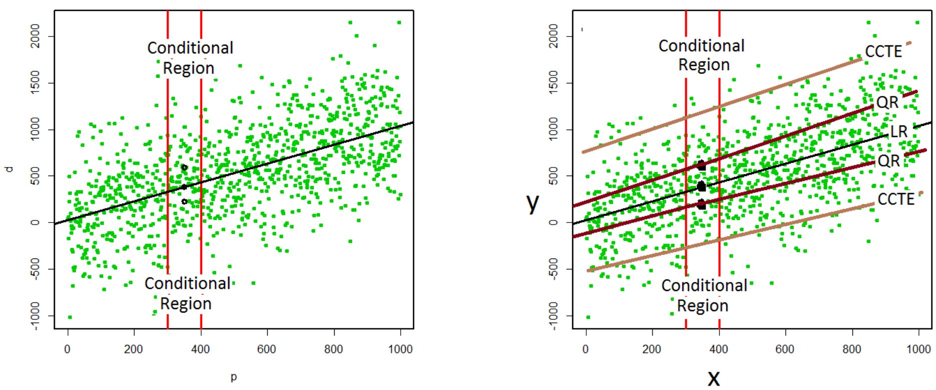
The regression model uses a standard least squares regression on a subset of the original data. That subset is determined from specific residuals of a quantile regression on the CTE target percentile. The use of QR methodology is in (Craighead, 2012). In that paper, Conditional VaR using QR is modeled and it was observed how various changes in economic scenario values led to insight on what directly affects the Conditional VaR amount at different percentiles.

This is well and fine as long as VaR is the target measurement. However, some capital and reserve models require an understanding of how economic values effect the average capital or reserves over (or under) the VaR limit. Currently, this is done by estimating the unconditional VaR by sorting the corporate model results and taking the average of all of the values above (or below) the VaR percentile and this process is called Tail Value at Risk (TVAR) or Conditional Tail Expectation (CTE). The term 'Conditional' here is related to the fact that you have to sort the results and take the worst results above or below the specific percentile of the VaR percent. So the CTE is conditional on the value of the VaR.

A standard linear regression is a model that relates the conditional mean of the observed variable to the predictors. Here 'conditional mean' is based on the mean of the observed variable related to a restricted region of the predictors. In the same way a quantile regression is the conditional VaR associated with a restricted subset of the predictors. Now if we regress on the subset of the original data, which is restricted to residuals above (or below) the Conditional Var limit, we have a Conditional 'Conditional Tail Expectation' (CCTE) regression model. For instance in Figure 3a, the regressions are conditioned on when x is between 300 and 400, which is labeled as the conditional region. Note the use of a two dimensional regression graphic to illustrate the concept of conditional regression. In Figure 3b we insert the various regression lines to demonstrate the left tail (bottom) and the right tail (top). The central line is the standard linear regression through the mean and the QR regressions are the quantile regression lines for some percentage above or below the mean. Then the bottom CCTE regression is a standard linear regression on where the residuals of the bottom quantile regression is below zero and the top CCTE regression is a standard linear regression when the residuals of the top quantile regression is above zero. Though not drawn, a separate linear regression that uses the intersection of where the

Figure 3: Associated graphs of the Capital

(a) Regression (b) Final Regessions



top quantile regression's residuals are below zero and the bottom quantile regression is greater than zero, produces a separate central regression line that excludes outliers. An outliers-removed regression may compete against other known robust regression methods, but requires further investigation.

A CCTE will never be as accurate as a CTE in the same fashion as a Conditional Var to the unconditional VaR, since the CTE is only conditional on the unconditional VaR and CCTE is dependent on the Conditional VaR. However, you can only develop a qualitative understanding of the impact of scenarios on CTE by examining a collection of best and worst case scenarios. With the CCTE you develop an actual quantitative understanding of the impact of the economic predictors on the conditional CTE.

These conditional VaR and CCTE models ease the development of dashboard models in Section 5.

In the analysis of corporate models, the need to observe the effect of an economic scenario on the model output (specifically economic capital for ERM models) gives the practitioner a critical understanding of the underlying economic risks contained in the model.

Observe the QR Conditional VaR formula

$$R_q = B_{0,q} + B_{1,q}X_1 + B_{2,q}X_2 + \cdots + B_{19,q}X_{19} + B_{20,q}X_{20} + U_q. \quad (1)$$

R_q is the capital response (specifically at the qth quantile), and the X_t are one of the risk drivers mentioned in Section 3 at the end of each year

t. The $B_{t,q}$ are the related coefficients for the specific quantile q and U_q is the error. The assumption that $Quant(U_q) = 0$ leads to the formula of the conditional quantile regression:

$$Quant(R_q) = B_{0,q} + B_{1,q}X_1 + B_{2,q}X_2 + \cdots + B_{19,q}X_{19} + B_{20,q}X_{20}. \quad (2)$$

In (Craighead, 2012) we examine various properties of Equations 1 and 2, especially around the use of Student-t statistics to determine significant coefficients associated with the predictors and other goodness-of-fit statistics, so please refer to that document for more detail on those components of the regression.

The mathematics that assures that $Quant(U_q) = 0$ gives the necessary foundation for the CCTE regression in that the residuals resulting from the fitting of Equation 2 creates the required subsets of the predictors and dependent variables. As mentioned above in the discussion of Figure 3b, this is done by using the quantile regressions residuals either above zero or below zero to properly subset the right tail or the left tail associated with the Conditional VaR. For instance, if you want to create the CCTE regression on the left tail, you will take all residuals of the quantile regression below or equal to zero and use those values to restrict the predictors and the respective responses. A standard linear regression on this subset will pass through the conditional means of the responses and result in a CCTE regression.

To distinguish CCTE regression formulas from the underlying quantile regression formulas use this:

$$CCTE(R_q; Orientation) = b_{0,q}+b_{1,q}X_1+b_{2,q}X_2+\cdots+b_{19,q}X_{19}+b_{20,q}X_{20} \quad (3)$$

where the $b_{i,q}$ denotes the coefficient for predictor X_i at time i. The value of $Orientation$ will be either $Left$, $Right$, $Bottom$ or Top. $Left$ and $Bottom$ will be based on negative or zero residuals and $Right$ and Top denote zero or positive residuals from the respective quantile regression $Quant(R_q)$.

The construction of various CCTE regressions will determine the effect of specific risk drivers from the underlying economic series in the scenarios. For instance, the example scenarios have both 90-day and 10-year Treasury rates, which allows one to understand how the change in the 10-year rate or the spread between the 10-year and 90-day rate influences the CCTE of the surplus. The creation of other esoteric risk drivers is easily done by using different functions on either or both of the underlying scenario series, if there is a specific need. Once the corporate model results are available and

Table 2: Quantile Regression results for 0.5% 10-year Treasury (Left tail)

Time	Coefficient	Standard Error	t value	$\Pr(>\|t\|)$	Significant Coefficient	Influence	Ranking
(Intercept)	1.280	0.475	2.693	0.007	NA	NA	NA
t1	92.396	33.170	2.786	0.005	92.396	0.094	F
t2	-98.147	35.513	-2.764	0.006	-98.147	0.100	E
t3	-247.593	38.232	-6.476	0.000	-247.593	0.253	A
t4	-160.356	30.839	-5.200	0.000	-160.356	0.164	C
t5	-180.837	29.024	-6.231	0.000	-180.837	0.185	B
t6	-143.592	37.979	-3.781	0.000	-143.592	0.147	D
t7	-56.589	28.809	-1.964	0.050	-56.589	0.058	G
t8	10.903	33.627	0.324	0.746	0.000	0.000	
t9	-27.137	33.062	-0.821	0.412	0.000	0.000	
t10	46.794	24.318	1.924	0.054	0.000	0.000	
t11	21.298	36.150	0.589	0.556	0.000	0.000	
t12	-39.627	29.036	-1.365	0.172	0.000	0.000	
t13	-26.421	26.572	-0.994	0.320	0.000	0.000	
t14	4.211	29.077	0.145	0.885	0.000	0.000	
t15	-17.096	29.446	-0.581	0.562	0.000	0.000	
t16	10.098	26.656	0.379	0.705	0.000	0.000	
t17	17.748	27.495	0.645	0.519	0.000	0.000	
t18	10.361	27.558	0.376	0.707	0.000	0.000	
t19	-24.687	28.713	-0.860	0.390	0.000	0.000	
t20	3.466	29.766	0.116	0.907	0.000	0.000	
Absolute Sum					979.511		

Table 3: CCTE Regression results for 0.5% 10-year Treasury (Left tail)

Time	Coefficient	Standard Error	t value	$\Pr(>\|t\|)$	Significant Coefficient	Influence	Ranking
(Intercept)	-0.881	0.344	-2.564	0.016	NA	NA	NA
t1	66.341	50.339	1.318	0.198	0.000	0.000	
t2	-124.612	57.306	-2.174	0.038	-124.612	0.163	D
t3	-189.754	46.663	-4.067	0.000	-189.754	0.248	C
t4	-228.764	61.855	-3.698	0.001	-228.764	0.299	A
t5	-221.112	79.442	-2.783	0.010	-221.112	0.289	B
t6	-57.141	74.401	-0.768	0.449	0.000	0.000	
t7	-67.863	35.288	-1.923	0.065	0.000	0.000	
t8	25.704	98.747	0.260	0.797	0.000	0.000	
t9	-52.805	91.394	-0.578	0.568	0.000	0.000	
t10	-5.664	59.803	-0.095	0.925	0.000	0.000	
t11	58.293	97.722	0.597	0.556	0.000	0.000	
t12	104.681	88.949	1.177	0.249	0.000	0.000	
t13	-132.457	88.222	-1.501	0.144	0.000	0.000	
t14	41.100	56.752	0.724	0.475	0.000	0.000	
t15	-15.922	79.978	-0.199	0.844	0.000	0.000	
t16	-80.222	80.459	-0.997	0.327	0.000	0.000	
t17	-40.565	84.702	-0.479	0.636	0.000	0.000	
t18	-50.173	61.575	-0.815	0.422	0.000	0.000	
t19	10.248	81.692	0.125	0.901	0.000	0.000	
t20	-38.776	56.797	-0.683	0.500	0.000	0.000	
Absolute Sum					764.242		

Table 4: Quantile Regression results for 99.5% 10-year Treasury (Right tail)

Time	Coefficient	Standard Error	t value	Pr(> |t|)	Significant Coefficient	Influence	Ranking
(Intercept)	30.111	0.036	839.051	0.000	NA	NA	NA
t1	316.130	2.335	135.416	0.000	316.130	0.198	A
t2	277.366	1.856	149.467	0.000	277.366	0.174	B
t3	215.893	1.929	111.921	0.000	215.893	0.136	C
t4	162.752	2.494	65.261	0.000	162.752	0.102	D
t5	109.761	2.388	45.968	0.000	109.761	0.069	E
t6	100.180	1.846	54.265	0.000	100.180	0.063	F
t7	62.215	2.302	27.032	0.000	62.215	0.039	G
t8	29.721	2.076	14.314	0.000	29.721	0.019	
t9	8.861	1.785	4.965	0.000	8.861	0.006	
t10	-1.118	3.194	-0.350	0.726	0.000	0.000	
t11	-21.232	3.029	-7.009	0.000	-21.232	0.013	
t12	-42.542	2.330	-18.255	0.000	-42.542	0.027	
t13	-46.397	3.455	-13.429	0.000	-46.397	0.029	
t14	-53.058	2.269	-23.388	0.000	-53.058	0.033	
t15	-53.757	1.979	-27.159	0.000	-53.757	0.034	
t16	-46.399	2.190	-21.191	0.000	-46.399	0.029	
t17	-17.927	1.797	-9.978	0.000	-17.927	0.011	
t18	-14.432	2.165	-6.667	0.000	-14.432	0.009	
t19	-11.283	1.838	-6.138	0.000	-11.283	0.007	
t20	-3.315	1.607	-2.063	0.039	-3.315	0.002	
Absolute Sum					1593.22		

Table 5: CCTE Regression results for 99.5% 10-year Treasury (Right tail)

Time	Coefficient	Standard Error	t value	Pr(> |t|)	Significant Coefficient	Influence	Ranking
(Intercept)	30.312	0.050	610.007	0.000	NA	NA	NA
t1	308.491	10.304	29.940	0.000	308.491	0.196	A
t2	275.186	9.578	28.731	0.000	275.186	0.175	B
t3	217.509	10.461	20.793	0.000	217.509	0.138	C
t4	162.299	10.086	16.091	0.000	162.299	0.103	D
t5	102.359	10.476	9.771	0.000	102.359	0.065	F
t6	105.700	7.752	13.635	0.000	105.700	0.067	E
t7	59.160	7.581	7.804	0.000	59.160	0.038	G
t8	21.818	7.002	3.116	0.004	21.818	0.014	
t9	-2.674	8.104	-0.330	0.744	0.000	0.000	
t10	0.168	7.706	0.022	0.983	0.000	0.000	
t11	-18.067	6.228	-2.901	0.007	-18.067	0.011	
t12	-54.432	6.199	-8.780	0.000	-54.432	0.035	
t13	-50.058	6.080	-8.233	0.000	-50.058	0.032	
t14	-55.290	6.403	-8.636	0.000	-55.290	0.035	
t15	-58.968	4.705	-12.532	0.000	-58.968	0.037	
t16	-44.631	4.595	-9.714	0.000	-44.631	0.028	
t17	-30.523	5.064	-6.027	0.000	-30.523	0.019	
t18	-7.314	5.217	-1.402	0.172	0.000	0.000	
t19	-10.920	3.452	-3.163	0.004	-10.920	0.007	
t20	-8.695	4.925	-1.766	0.089	0.000	0.000	
Absolute Sum					1575.412		

Table 6: Quantile Regression results for 70.0% 10 Year Rate (Right tail)

Time	Coefficient	Standard Error	t value	Pr($>\lvert t\rvert$)	Significant Coefficient	Influence	Ranking
(Intercept)	27.631	0.036	758.796	0.000	NA	NA	NA
t1	217.551	2.493	87.248	0.000	217.551	0.172	A
t2	183.612	2.813	65.277	0.000	183.612	0.146	B
t3	130.289	3.015	43.218	0.000	130.289	0.103	C
t4	92.061	3.005	30.641	0.000	92.061	0.073	D
t5	50.763	3.267	15.540	0.000	50.763	0.040	
t6	27.001	3.319	8.134	0.000	27.001	0.021	
t7	-9.475	3.531	-2.683	0.007	-9.475	0.008	
t8	-30.168	3.619	-8.337	0.000	-30.168	0.024	
t9	-40.999	3.939	-10.407	0.000	-40.999	0.032	
t10	-47.079	4.118	-11.434	0.000	-47.079	0.037	
t11	-60.134	4.071	-14.770	0.000	-60.134	0.048	
t12	-79.347	4.045	-19.616	0.000	-79.347	0.063	E
t13	-70.841	3.995	-17.732	0.000	-70.841	0.056	F
t14	-66.563	3.859	-17.248	0.000	-66.563	0.053	G
t15	-66.034	4.125	-16.007	0.000	-66.034	0.052	
t16	-47.963	3.919	-12.237	0.000	-47.963	0.038	
t17	-25.408	3.625	-7.009	0.000	-25.408	0.020	
t18	-16.598	3.857	-4.303	0.000	-16.598	0.013	
t19	-6.535	3.749	-1.743	0.081	0.000	0.000	
t20	4.945	3.067	1.612	0.107	0.000	0.000	
Absolute Sum					1261.889		

Table 7: CCTE Regression results for 70.0% 10 Year Rate (Right tail)

Time	Coefficient	Standard Error	t value	Pr($>\lvert t\rvert$)	Significant Coefficient	Influence	Ranking
(Intercept)	28.643	0.017	1655.513	0.000	NA	NA	NA
t1	252.669	2.908	86.889	0.000	252.669	0.187	A
t2	212.891	2.816	75.590	0.000	212.891	0.158	B
t3	164.315	2.749	59.779	0.000	164.315	0.122	C
t4	120.254	2.742	43.864	0.000	120.254	0.089	D
t5	73.760	2.610	28.265	0.000	73.760	0.055	E
t6	52.000	2.538	20.493	0.000	52.000	0.039	
t7	14.096	2.512	5.611	0.000	14.096	0.010	
t8	-10.270	2.290	-4.485	0.000	-10.270	0.008	
t9	-22.175	2.283	-9.714	0.000	-22.175	0.016	
t10	-35.309	2.180	-16.196	0.000	-35.309	0.026	
t11	-47.195	2.072	-22.776	0.000	-47.195	0.035	
t12	-65.795	2.030	-32.412	0.000	-65.795	0.049	G
t13	-64.627	1.938	-33.352	0.000	-64.627	0.048	
t14	-66.081	1.848	-35.762	0.000	-66.081	0.049	F
t15	-59.486	1.831	-32.490	0.000	-59.486	0.044	
t16	-42.634	1.783	-23.909	0.000	-42.634	0.032	
t17	-25.113	1.732	-14.500	0.000	-25.113	0.019	
t18	-12.592	1.682	-7.488	0.000	-12.592	0.009	
t19	-7.590	1.662	-4.568	0.000	-7.590	0.006	
t20	2.332	1.634	1.427	0.154	0.000	0.000	
Absolute Sum					1348.853		

the scenarios contained within the modeling environment (in this situation the R environment), it is very simple to generate various regressions that reveal relationships without having to rerun the corporate model. This way the extractation of additional information from the stochastic results is done with a minimal amount of effort. Section 5 discusses how to use regression models as dashboards to monitor material effects as the exterior economy changes.

In the example below, there will only be one economic series, which is the change in the ten-year Treasury rate through time. The risk driver does not have to be a single series through time. The risk driver chosen can have any relationship between different series such as the spread between the ten-year note rate and the 90-day bill rate. However, note that these series are consistent through time. If the drivers have different attributes at different times, the approach of using the ranking of the absolute value of the coefficients becomes invalid, since this requires the predictors to be on a consistent basis. However, if the dashboard requires looking at multiple risk drivers that are not consistent between themselves, the need to standardize the predictors arises to make the regressions comparable between the various risk drivers. For instance if the scenario set has both interest rates and equity indices, by standardizing, the mean and variance of the separate interest rate series will be on a consistent basis as the standardized equity series and the use of rank ordered absolute coefficients becomes valid.

Since CCTE regression builds upon a quantile regression, we look at the some of the quantile regressions from (Craighead, 2012) and construct the CCTE regressions on those. Tables 2 and 4 have the same coefficients as in (Craighead, 2012) but small improvements on the Student-t values lead to small changes in the original influence ranking. The resultant CCTE regressions are in tables 3 and 5.

These models, developed through the methods in Appendix A, reveal the relevant information that is needed for the practitioner. Initially, the actual value of the CCTE coefficients is not as critical to the understanding as is the relative magnitude when compared to all of the coefficients. The use of the absolute magnitude of the coefficients locates the year of a specific risk driver as defined in the design matrix of the regression. This approach takes on a qualitative nature in that it does not predict the actual CTE values, but it is used to see what influences risk or profit. The pricing actuary can use the qualitative averages approach to determine design flaws when examining the averages of values below low quantiles and positive upside design features in averages over high quantiles. The valuation actuary can use this type of report to locate various risks and locations of those risks in existing lines of business. This also allows the actuary and the financial engineer to determine risk exposure from embedded options in the business. The financial engineer can also use these methods to improve his or her derivative hedge. In the past the practitioner may have used different deterministic scenarios to determine the direction of the markets that created risk exposure to the business. The deterministic scenarios do not indicate the significance or aid in the determination of the exact location in the projection period that the business is at the highest risk.

Note the following relationship for $CCTE(R_q; Right)$ to the various risk drivers X_t. If value of the X_t can have both positive and negative values, we need only to examine the large $|b_{t,q}|$. If one is studying where average profit is enhanced at the specific percentage being studied, if X_t is positive and $|b_{t,q}|$ is large and $b_{t,q}$ is positive, $CCTE(R_q; Right)$ increases. If X_t is negative and $b_{t,q}$ is negative, $CCTE(R_q; Right)$ also increases. Just reverse this reasoning if one is interested in determining when the business model is at a risk for loss, which is the consideration of $CCTE(R_q; Left)$.

In Table 3 we display the CCTE model regression that corresponds to the average capital below (left tail or bottom residual in the conditional region) the 0.5% target percentile of the EVAS as modeled against the change in 10-year Treasuries risk driver Y_t^{10} as mentioned in Section 3.

In this table, the values in the coefficient column correspond to the $b_{t,.005}$ in Equation 3. The Standard Error column displays confidence bounds on each coefficient's estimate. The t–value column displays the value of the Student-t statistic relating the standard error to the coefficient's value. A good rule of thumb for coefficient significance is for the absolute value of the Student-t statistic to be greater than two. The associated probability with the Student-t statistic is the $Pr(> |t|)$ column. If $Pr(> |t|)$ is less than 5%, then we can be at least 95% confident in the estimate of the coefficients. These five columns are direct output from the `rq` function in R in Appendix A. The additional columns display the impact of the coefficients on the model and are generated by the `signcoeff` function defined in the Appendix. The Significant Coefficient column is the absolute value of the $b_{t,.005}$ if the Student-t statistic is significant, or zero if it is not. The Absolute Sum of this column is then used to derive the Influence value by taking a specific Significant Coefficient value divided by the Absolute Sum. The Influence values denote at what point in time the risk driver has influence and the amount of influence on the CCTE regression as in (Craighead, 2012). The Ranking column displays the top Influence values. So, an A ranking will be the location in time that the underlying risk driver has the most influence on the CCTE regression. Its overall contribution to the CCTE model is equal converting the Influence value to percent.

In the analysis each risk driver through time must have the same underlying characteristics. So, the Intercept coefficient is excluded in the analysis below, but if a CCTE model is implemented as a dashboard model the Intercept should be included.

The Significant Coefficient column only has non-zero values if the t probability is less than 5%. The absolute sum of the coefficients are at the bottom of this column for comparison purposes.

As mentioned above, the effectiveness of the influence formula holds if the underlying X_t are of similar magnitude. For instance, this approach does not work if a risk driver is the combination of a time series of interest rate changes and a time series of changes in equity returns. Since the change in interest rates is less volatile than that of the change in equity returns, larger coefficients arise from the interest rate changes than from the coefficients associated with the equity changes.

The Ranking column is just an alphabetical ranking to further distinguish which time in the future the risk driver has the greatest impact.

Now to interpret the CCTE models, look at the change in 10-year rates risk driver Y_t^{10} in Table 3.

- The 0.5% model corresponds to severe downside possibilities. For instance, year 4 has the most impact on the downside risk, since it has a large negative coefficient and if there is a large positive change in the 10-year rate, the model indicates that things will worsen. From the Influence value, we see that this one coefficient explains 29.9% of the change in the model. In addition, note that if the change in the 10-year rates are increasing in years 2 through 5, the company is at increased risk.

- Look at the 99.5% upside model in Table 5, notice how the largest significant coefficient starts at year 1 and the significance is high through year 7. So if the change in the 10-year rate is sharply increasing in these years, we should see positive increases in the model, which means that the conditional average capital will grow.

- If you compare the left tail quantile regression in Table 2 to that of the CCTE regression you will observe that the drivers for the CCTE are less complex than the quantile regression. However, when you compare the right tail quantile regression in Table 4 the rankings are almost identical with that of the right tail CCTE regression. This is an example where analyzing both quantile and CCTE regressions give additional insight on the underlying risk driver.

The use of QR and CCTE regressions allow the practitioner to conduct risk analysis on several different risk measures. In fact in the past the practitioner did not consider some of the above analyses without extensive additional computer runs. This increased ability may initially raise more questions for the practitioner to analyze, but this type of risk analysis is an excellent tool to conduct these analyses.

Next we apply CCTE regression to determine the 70% CTE on the data to demonstrate CCTE regression as an additional analysis tool to use in PBR.

4.1 Principle-Based Reserves

Regulatory control, arising from the prior meltdown, has increased the use of stochastic scenarios in the determination of reserves through the use of principle based approaches. The development of the revised standard valuation law to allow for PBR and life principle-based reserves under Valuation Manual 20 (VM-20) and Actuarial Guidelines 43 (AG-43) are examples of the use of CTE methodology in the determination of stochastic reserves.

Since principle based reserves is the maximum of the gross premium reserve, the deterministic reserve and the stochastic reserve, as long as the gross premium or the deterministic reserve exceeds the stochastic reserve, there is not much need to consider what influences the stochastic reserve. We are assuming that the stochastic reserve is a 70% CTE estimate. But, if a product has embedded options, the stochastic reserve can dominate the other reserves. In this situation PBR can add volatility to the balance sheet and to the GAAP earnings. In that situation, having a dashboard that estimates the impact of frequent economic changes on the stochastic reserve is useful. Even if the stochastic reserve does not dominate the other reserves, CCTE regression determines what conditions influence the growth of the reserve and this is informative to the valuation or pricing actuary.

We reuse the capital data to analyze how the change in 10-year Treasury rates influence the $CCTE(R_{70}; Right)$ model. Tables 6 and 7 are the respective quantile and CCTE regressions.

We see that both the CCTE and the quantile regressions have similar Rankings where the change of the ten-year rate in year one has the highest influence on the regression. It does have more influence on the CCTE regression than the quantile regression where its influence is one percent higher, with an influence of 18.7% versus 17.2%. With strictly positive coefficients for the first five years, we see that if the ten-year rates increase over that period where the resulting $Quant(R_{70})$ and $CCTE(R_{70}; Right)$ increase and thereby the stochastic reserves increase. The reserves are also sensitive if the magnitude of the change in rates is large, since the coefficients are large. So a sharply rising interest rate environment increases the reserves. Now, observe in both regressions that in years 12 through 14, that the coefficients are negative, but their absolute value is between $\frac{1}{2}$ and $\frac{1}{3}$ in magnitude to the positive coefficients in the first five years. In this situation if rates continue to rise in years 12 through 14, then the reserves will decline, but if rates fall instead, then the reserves will go up. The worst case scenario is for rates to sharply rise in the first seven years and then sharply fall from year 8 forward. The 70th percentile has this same sensitivity, but less so. Here we see that the actual reserves determined by the average of the worst cases over the 70th percentile are more sensitive to the economy than the 70th percentile alone.

In the analysis, observe the construction of a scenario that tasks the reserves. Possibly, a closer use of the regression could refine this type of scenario further. In corporate Valuation and ERM, the determination of sets of extreme scenarios to target specific limits for product lines and accumulated values from those lines are rarely in common. There is a need to conduct further study to determine the feasibility of using regression in common extreme scenario design.

In the next section the use of CCTE regression in dashboard construction is reviewed.

5 Dashboard model construction

Below is an outline of turning QR and CCTE results into dashboards:

- Pick a specific risk driver based on the scenarios, which can be easily extracted from current daily or weekly economic data.

- Choose the VaR target percent.
- Produce the related QR and CCTE model on the specific risk driver.
- Use a technique to approximate future values of the economic indicator. For example, if the risk driver is related to the change in 10-year Treasuries, take the current yield curve and produce the implied 10-year forward rates at times where the coefficients are significant. Using these forward rates, then replace the predictors with the change of rates between the separate 10-year forwards.

To model spreads, create the 90-day forward rates and calculate the spread at each time in the future.

If the risk driver is an equity return there are two approaches to the construction of the dashboard. One, assume that the current economic return is held constant into the future due to a no arbitrage assumption, and all of the predictors in the model is replaced with that single value. Another approach is to actually use a simple economic generator for that equity return and produce multiple equity scenarios and quickly process these future returns through the model and average the results, or look at the evolving uncertainty through time.

If the risk driver is either a change in call prices, put prices or equity volatilities, take a similar simulation approach for equity returns.

6 General Comments, Conclusions and Future Research

(Koenker and Machado, 1999), (Portnoy, 1999), and (Craighead, 2000) discuss several ways to display Quantile Regression results. These can also be used to display CCTE regressions as well.

Below is a list of strengths and weaknesses of this methodology.

6.1 Strengths and Weaknesses

The strengths of the CCTE methodology are:

- The input scenarios tie to the output.

- The sign and magnitude of the coefficients give insight into risk exposures.
- Averages above or below specific percentiles are targets in the output.
- The model reveals the influence of a specific period in time to the capital for a specific risk driver.
- Models can be calibrated very fast. The regressions on 10,000 scenarios are usually under one minute in *R*.
- One can use standard regression analytics on the CCTE regressions.
- The models allows for quick sensitivity testing.
- Though the examples are linear models, you can conduct non-linear regressions on the subsets of residuals.
- The analysis can be conducted at separate levels. For instance, CCTE can be used to examine the impact of changing economics on a company's reserves or its capital, if a company uses principle based approaches to estimate its economic capital.

The main weaknesses of the use of CCTE are

- It is relatively complex, since it is layered on a quantile regression's results.
- Close scrutiny is required to not oversimplify the impact of specific risk drivers on the capital models.
- Extreme outliers do affect the results since the CCTE regression is a standard linear regression.

6.2 Concluding Remarks

This paper has outlined the development of the CCTE methodology. This methodology has developed a report that reveals the impact of a risk driver at specific times and provides another quantitative approach to understanding the behavior of business. Also, this paper has examined the use of CCTE in the analysis of the stochastic reserve sensitivity to a risk driver. Also, the development of using CCTE in the design of dashboards was addressed, especially in the monitoring of EC and the stochastic component of principle based reserves and capital. Future research will be conducted on other applications of the CCTE regression that require averaging values between specific conditional regions. In addition, how effective a CCTE regression is as a robust regression, since it can easily allow the exclusion of outliers within the data will be examined. Also this new regression shows promise in scenario design, and we will expand this methodology in that area as well.

A Linear and Quantile Regression Modeling in R

R based on (R Development Core Team, 2017) has become the lingua franca of the statistical world. Though most of the analysis from ERM models occurs in Excel, R is still a good candidate to conduct extensive statistical analyses with the related graphical output. Some of R's benefits are:

- It is an open source system.
- It runs on multiple platforms.
- It is free.
- It can easily be integrated into multiple packages including Excel.
- It is constantly improving with cutting edge statistical tools being developed by researchers.
- Leading subject matter experts, such as Koenker, have created and continue to maintain high quality packages that can used by anyone willing to learn a new computer language.

Below, the economic scenarios and the corporate model EVAS results are within R and we outline how to conduct the various regressions. To simplify the use of significant coefficients the R function `singcoeff` is below. Its design allows its application to both quantile and linear regressions, where it converts a standard regression report into a data frame that contains the significant coefficients along with the ranking of the influence of those coefficients.

```
signcoeff<-function(regobj,reg=1)
{
if(class(regobj)=="lm") x<-summary(regobj)$coefficients
if(class(regobj)=="rqs") x<-summary(regobj,se="iid")[[reg]][[3]]
signcoef<-as.numeric(x[,4]<.05)*x[,1]
influence<-signcoef[2:length(signcoef)]
inf<-abs(influence)/sum(abs(influence))
qqq<-c(0,inf)
qqqq<-factor(rank(c(-qqq)))
qqqq
noleves<-length(levels(qqqq))
if(noleves==1) levels(qqqq)<-""
if(noleves==2) levels(qqqq)<- c("A","")
if(noleves==3) levels(qqqq)<- c("A","B","")
if(noleves==4) levels(qqqq)<- c("A","B","C","")
if(noleves==5) levels(qqqq)<- c("A","B","C","D","")
if(noleves==6) levels(qqqq)<- c("A","B","C","D","E","")
if(noleves==7) levels(qqqq)<- c("A","B","C","D","E","F","")
if(noleves==8) levels(qqqq)<- c("A","B","C","D","E","F","G","")
if(noleves>=9) levels(qqqq)<- c("A","B","C","D","E","F","G",rep("",noleves-7))
qqqq
signcoef[1]<-0.0
inf<-c(0.0,inf)
xxx<-data.frame(cbind(x,signcoef,inf,as.character(qqqq)))
names(xxx)<-c(colnames(x),"Significant\nCoeffient","Influence\nPercent","Ranking")
xxx[,1:6] <- lapply(xxx[,1:6], function(x) as.numeric(as.character(x)))
xxx
}
```

Below are the commands for the QR and CCTE regressions on the change in the 10-year Treasury rate study:

```
library(quantreg)
rqcase<-data.frame(cbind(LOB,nmrs10[,2:21]))
names(rqcase)[1]<-"V1"
rq10<-rq(V1~.,data=rqcase,tau=c(.005,.995),method="fn")
summary(rq10)
signcoeff(rq10)
rq10one<-as.numeric(names(rq10$residuals[rq10$residuals[,1]<=0,1]))
rq10low<-lm(V1~.,data=rqcase[rq10one,])
summary(rq10low)
signcoeff(rq10low)
signcoeff(rq10,2)
```

```
rq10two<-as.numeric(names(rq10$residuals[rq10$residuals[,2]>=0,2]))
rq10high<-lm(V1~.,data=rqcase[rq10two,])
summary(rq10high)
signcoeff(rq10high)
```

The first command loads the `quantreg` package into R.

The second line creates the data frame. The term `evasadj[,39]` references the EVAS data frame for the 39th line of business's values and the `nmrs10[,2:21]` data frame is the 10-year Treasury rates from time 1 through 20. Since time 0 rates are all the same, if you include this rate, the regressions will fail, since the time 0 predictor is not independent from the time 1 through time 20 predictors. The capital and the 10-year Treasuries rates are combined into one data frame using `cbind` and `data.frame` commands. The results are stored in `rqcase`.

The `names` command is used to assure that the capital value has a consistent name of `V1`.

The fourth command is where the actual QR model is built by the use of the `rq` function. Using the model formula framework, the first variable in the data frame is named `V1` (which is the EVAS) is modeled against all of the other variables in the data frame by the use of the `V1~.` command. The data frame is referenced by the `data =` command and the 0.5% and 99.5% quantiles are input by the `tau=c(.005,.995)` command. The method of fitting indicated by the `method="fn"` command specifies the Frisch–Newton interior point method. Finally the model is stored into the QR object `rq10`.

The summary command produces QR results similar to these:

```
Call: rq(formula = V1 ~ ., tau = c(0.005, 0.995), data = rqcase, method = "fn")

tau: [1] 0.005

Coefficients:
            Value      Std. Error t value    Pr(>|t|)
(Intercept)    1.28037    0.47549    2.69276    0.00710
t1            92.39614   33.16990    2.78554    0.00535
t2           -98.14702   35.51337   -2.76366    0.00573
t3          -247.59339   38.23197   -6.47608    0.00000
t4          -160.35555   30.83934   -5.19971    0.00000
t5          -180.83712   29.02385   -6.23064    0.00000
t6          -143.59239   37.97919   -3.78082    0.00016
t7           -56.58887   28.80896   -1.96428    0.04953
t8            10.90279   33.62723    0.32422    0.74577
t9           -27.13700   33.06195   -0.82079    0.41178
t10           46.79366   24.31780    1.92426    0.05435
t11           21.29753   36.14966    0.58915    0.55577
t12          -39.62687   29.03554   -1.36477    0.17236
t13          -26.42105   26.57228   -0.99431    0.32010
t14            4.21118   29.07729    0.14483    0.88485
t15          -17.09598   29.44618   -0.58058    0.56153
t16           10.09816   26.65600    0.37883    0.70482
t17           17.74801   27.49546    0.64549    0.51863
t18           10.36056   27.55791    0.37596    0.70696
```

```
t19         -24.68656   28.71318   -0.85976    0.38994
t20           3.46574   29.76639    0.11643    0.90731

Call: rq(formula = V1 ~ ., tau = c(0.005, 0.995), data = rqcase, method = "fn")

tau: [1] 0.995

Coefficients:
            Value     Std. Error t value   Pr(>|t|)
(Intercept)  30.11057   0.03589  839.05051   0.00000
t1          316.12966   2.33451  135.41568   0.00000
t2          277.36559   1.85570  149.46708   0.00000
t3          215.89263   1.92897  111.92126   0.00000
t4          162.75161   2.49385   65.26109   0.00000
t5          109.76131   2.38780   45.96755   0.00000
t6          100.17991   1.84611   54.26537   0.00000
t7           62.21496   2.30157   27.03154   0.00000
t8           29.72095   2.07635   14.31403   0.00000
t9            8.86119   1.78462    4.96531   0.00000
t10          -1.11805   3.19430   -0.35001   0.72634
t11         -21.23153   3.02918   -7.00899   0.00000
t12         -42.54223   2.33043  -18.25512   0.00000
t13         -46.39661   3.45490  -13.42921   0.00000
t14         -53.05779   2.26862  -23.38773   0.00000
t15         -53.75742   1.97937  -27.15890   0.00000
t16         -46.39921   2.18953  -21.19139   0.00000
t17         -17.92702   1.79668   -9.97785   0.00000
t18         -14.43172   2.16469   -6.66687   0.00000
t19         -11.28290   1.83833   -6.13757   0.00000
t20          -3.31534   1.60703   -2.06302   0.03914
```

The `signcoeff` command updates the first quantile regression above with the significant coefficients results. The results are:

```
                  Value Std. Error    t value     Pr(>|t|) Significant\nCoeffient Influence\nPercent Ranking
(Intercept)    1.280373  0.4754869  2.6927611 7.098137e-03                0.00000          0.00000000
t1            92.396145 33.1699047  2.7855415 5.353921e-03               92.39614          0.09432890       F
t2           -98.147016 35.5133704 -2.7636638 5.726119e-03              -98.14702          0.10020007       E
t3          -247.593392 38.2319656 -6.4760832 9.856738e-11             -247.59339          0.25277258       A
t4          -160.355548 30.8393418 -5.1997072 2.035563e-07             -160.35555          0.16370988       C
t5          -180.837117 29.0238517 -6.2306381 4.832936e-10             -180.83712          0.18461989       B
t6          -143.592393 37.9791863 -3.7808181 1.572254e-04             -143.59239          0.14659608       D
t7           -56.588874 28.8089565 -1.9642806 4.952528e-02              -56.58887          0.05777261       G
t8            10.902787 33.6272268  0.3242250 7.457745e-01                0.00000          0.00000000
t9           -27.136996 33.0619455 -0.8207925 4.117841e-01                0.00000          0.00000000
t10           46.793662 24.3178047  1.9242552 5.435101e-02                0.00000          0.00000000
t11           21.297531 36.1496568  0.5891489 5.557747e-01                0.00000          0.00000000
t12          -39.626875 29.0355380 -1.3647715 1.723556e-01                0.00000          0.00000000
t13          -26.421055 26.5722765 -0.9943090 3.200965e-01                0.00000          0.00000000
t14            4.211179 29.0772877  0.1448271 8.848503e-01                0.00000          0.00000000
t15          -17.095976 29.4461792 -0.5805838 5.615341e-01                0.00000          0.00000000
t16           10.098163 26.6560036  0.3788326 7.048202e-01                0.00000          0.00000000
t17           17.748007 27.4954631  0.6454886 5.186255e-01                0.00000          0.00000000
t18           10.360561 27.5579076  0.3759560 7.069576e-01                0.00000          0.00000000
t19          -24.686560 28.7131769 -0.8597641 3.899397e-01                0.00000          0.00000000
t20            3.465736 29.7663911  0.1164312 9.073132e-01                0.00000          0.00000000
```

The development of `rq10one` variable is finding which predictors and results are causing the first quantile regression's residuals to be negative or zero. This is the key point of being able to create the CCTE regression, since the point of interest is where the quantile regression solves for predictors when the capital values fall below or on the conditional VaR. The `rq10one` variable determines the subset of `rqcase` we conduct the linear regression on. Since the linear regression targets only these values and the mean of the dependent variable, then that regression is a CCTE regression.

The next row with `rq10low` actually creates the CCTE regression on the subset of the data from `rq10one`.

The `summary(rq10low)` command produces the summary of the linear regression. This is not displayed due to space limitations.

The `signcoeff(rq10low)` command produces this result:

	Estimate	Std. Error	t value	Pr(>|t|)	Significant\nCoeffient	Influence\nPercent	Ranking
(Intercept)	-0.8808962	0.3435106	-2.56439295	0.0159882235	0.0000	0.0000000	
t1	66.3410442	50.3385565	1.31789723	0.1982193206	0.0000	0.0000000	
t2	-124.6117134	57.3060985	-2.17449306	0.0382740157	-124.6117	0.1630527	D
t3	-189.7539432	46.6625299	-4.06651640	0.0003514775	-189.7539	0.2482903	C
t4	-228.7640759	61.8548677	-3.69840054	0.0009374841	-228.7641	0.2993345	A
t5	-221.1124330	79.4421582	-2.78331352	0.0095302118	-221.1124	0.2893225	B
t6	-57.1405005	74.4011974	-0.76800512	0.4489130971	0.0000	0.0000000	
t7	-67.8629088	35.2884686	-1.92309022	0.0646928572	0.0000	0.0000000	
t8	25.7037046	98.7472288	0.26029798	0.7965386238	0.0000	0.0000000	
t9	-52.8052056	91.3942012	-0.57777414	0.5680343805	0.0000	0.0000000	
t10	-5.6639279	59.8026295	-0.09471035	0.9252194404	0.0000	0.0000000	
t11	58.2931129	97.7222344	0.59651842	0.5556216186	0.0000	0.0000000	
t12	104.6812824	88.9486762	1.17687285	0.2491542953	0.0000	0.0000000	
t13	-132.4567447	88.2218087	-1.50140591	0.1444446968	0.0000	0.0000000	
t14	41.1001081	56.7522854	0.72420182	0.4749499136	0.0000	0.0000000	
t15	-15.9223788	79.9782818	-0.19908378	0.8436358388	0.0000	0.0000000	
t16	-80.2223340	80.4590631	-0.99705777	0.3272755803	0.0000	0.0000000	
t17	-40.5645552	84.7015499	-0.47891160	0.6357204551	0.0000	0.0000000	
t18	-50.1730405	61.5745862	-0.81483358	0.4220444267	0.0000	0.0000000	
t19	10.2480407	81.6921486	0.12544707	0.9010660162	0.0000	0.0000000	
t20	-38.7759944	56.7972389	-0.68270915	0.5004005765	0.0000	0.0000000	

The summary statement `summary(rq10)` gives the quantile regression summary from both tails. However, when using the `signcoeff` function on the quantile regression object you have to specific which regression by setting the optional variable `reg` to correspond to which regression. So `signcoeff(rq10,2)` displays the significant coefficients summary on the right hand tail for the 99.5% conditional VaR.

The development of the `rq10two` variable is to pick all residuals above and equal to zero to properly capture the conditional data from the quantile regression. This variable is then used in the linear regression `lm` function to subset the input data to restrict the linear regression to just the right tail data. The linear regression results is stored in the `rq10high` object. The standard regression results on the right tail CCTE regression is produced by using `summary(rq10high)` and the corresponding significant coefficients results come from using the `signcoeff(rq10high)` function.

To standardize a regression we use the `scale` function to scale one variable and use the `lapply` with `scale` to scale all variables or selective ones, for instance `rqcase1<-lapply(rqcase,scale)` rescales all of the data in the `rqcase` data frame above.

B Disclaimer

The design, use and conclusions of this paper and its related research reflect only the personal opinions of the author. It does not reflect any opinions or positions of Columbia University, Pacific Life Insurance, Nationwide Insurance, or the Society of Actuaries.

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