Recent adult mortality trends in Canada, the United States and other low mortality countries

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Abstract

This paper examines recent changes in the age-at-death distribution at older ages in Canada, the U.S. and eight additional low-mortality countries. Data starting in 1950 are taken from the Human Mortality Database and a flexible two-dimensional smoothing approach based on P-splines is used to monitor these changes. The U.S. displayed the most worrying picture for the latest two decades. Indeed, for several consecutive years in that timeframe, US females and males have both recorded important declines in their modal age at death and their level of old-age mortality remains high compared to the other countries. Thus, although Canada and the U.S. are neighbouring countries, the findings for the former regarding recent old-age mortality trends rather resemble those obtained for the remaining eight low mortality countries studied. Further analysis of changes in the age-at-death distribution at older ages by socioeconomic group or by region could improve our current understanding of the latest mortality dynamics recorded among US adults.

INTRODUCTION

Background

The 20th century has been an important milestone in the evolution of epidemiological conditions in today's low mortality countries. With the fall in infant mortality, infectious and parasitic diseases, and more recently cardiovascular diseases and cancer, the probability to survive to older ages increased substantially. As the probability to survive to older ages increases over time within human populations, life expectancy at birth rises and the shape of the survival curve usually progressively becomes more rectangular. This phenomenon, well-known as the *rectangularization of the survival curve*, is associated with a reduction in the variability of age at death (Wilmoth and Horiuchi, 1999), commonly referred to as the *compression of mortality*. Indeed, when compression of mortality is at work, deaths are concentrated into a shorter age interval over time, the dispersion of the age-at-death distribution is reduced, and the downward slope of the survival curve becomes steeper, resulting in a more rectangular shape.

An abundant body of demographic and epidemiology literature has already focussed on the topics of rectangularization of the survival curve and compression of mortality, providing evidence of these phenomenon in several low mortality countries and regions (Fries, 1980; Myers and Manton, 1984a,b; Nagnur, 1986; Manton and Tolley, 1991; Hill, 1993; Eakin and Witten, 1995; Nusselder and Mackenbach, 1996, 1997; Paccaud et al., 1998; Wilmoth and Horiuchi, 1999; Kannisto, 2000, 2001, 2007; Lynch and Brown, 2001; Robine, 2001; Martel, 2002; Martel and Bourbeau, 2003; Cheung et al., 2005, 2008, 2009; Cheung and Robine, 2007; Ouellette and Bourbeau, 2009; Thatcher et al., 2010). These studies also suggest that roughly up until the 1950s, important mortality reductions among infants, children and even young adults led to a strong compression of the overall age-at-death distribution in the various countries and regions over time. Afterwards however, this overall compression of mortality slowed down substantially, even though important mortality gains started to be recorded consistently among older adults (Kannisto et al., 1994; Jeune and Vaupel, 1995).

As pointed out by Thatcher et al. (2010), these findings have encouraged researchers to distinguish *old-age* mortality compression from *overall* mortality compression, and to study changes in the age-at-death distribution at *older ages* and over the *entire age range* separately. Moreover, the adult modal age at death (referred to hereafter as the modal age at death) and the variability of deaths around this modal age emerged as an important set of tools to summarize and monitor changes in the age-at-death distribution at older ages over time (Kannisto, 2000, 2001, 2007; Cheung et al., 2005, 2008, 2009; Cheung and Robine, 2007; Canudas-Romo, 2008, 2010; Ouellette and Bourbeau, 2009; Thatcher et al., 2010). Indeed, unlike life expectancy at birth, which is highly sensitive to changes in mortality among infants and children, the modal age at death is solely influenced by adult mortality and consequently much more sensitive to changes occurring among the elderly population (Kannisto, 2001; Horiuchi, 2003; Cheung and Robine, 2007; Canudas-Romo, 2010).

Study objectives

The latest studies focusing specifically on changes in the distribution of ages at death at older ages underline that some of today's lowest mortality countries might have taken on new paths where, notably, deaths above the modal age at death no longer tend to concentrate into a narrower age interval over time (Cheung et al., 2008, 2009; Cheung and Robine, 2007; Ouellette and Bourbeau, 2009). The shifting mortality regime first introduced by Kannisto (1996) and Bongaarts and Feeney (2002, 2003), and furthered by Bongaarts (2005) under which adult mortality is assumed to shift to higher ages over time has been identified as a plausible successor. Nevertheless, deviations from this regime have been noticed and further research on the topic is needed.

Accordingly, the first objective of this paper is to present a flexible two-dimensional smoothing approach based on P-splines which has the potential to refine our monitoring of recent changes in the age-at-death distribution that have occurred at older ages. This approach generalizes the one developed by Ouellette and Bourbeau (2009) where calendar years were treated independently from one another. Then, the second objective of the present paper is to apply this new approach to Canadian and US data in order to compare their recent trends in adult mortality. Comparisons with other low mortality countries, namely Denmark, England and Wales, France, Italy, Japan, the Netherlands, Sweden and Switzerland, are also provided.

METHODS

Data

The data for this study were taken from the well-known and widely used Human Mortality Database (HMD, 2010). For each country included in this analysis, we extracted observed deaths counts and exposure data by sex, single year of age, and single calendar year starting from 1950. Data below age ten were not extracted given our aim to focus on adult mortality. Furthermore, data selected for France (1950-2007) and England and Wales (1950-2006) refer to the civilian population, while those for Canada (1950-2006), Denmark (1950-2007) Italy (1950-2006), Japan (1950-2007), the Netherlands (1950-2006), Sweden (1950-2007), Switzerland (1950-2007) and the U.S. (1950-2006) refer to the total population.

Smoothing mortality data with P-splines

Recently, Ouellette and Bourbeau (2009) introduced the use of a one-dimensional smoothing approach, usually known as the method of P-splines (Eilers and Marx, 1996), to monitor with great precision changes in the adult age-at-death distribution over time in low mortality countries. As opposed to common parametric approaches to model the age pattern of mortality, for example the Gompertz, logistic, and Siler models, the method of P-splines does not rely on any rigid theoretical assumptions or modelling structure. Such flexibility leads to a finer expression of the underlying mortality trend over age, as described by the actual data. It consequently refined the monitoring of alterations in the ages-at-death distribution over time.

Since mortality change over age and over time is expected to be regular over both ages and calendar years, this paper provides a generalization of this one-dimensional smoothing method. Indeed, we introduce the use of a two-dimensional smoothing approach (Camarda, 2008, 2009; Currie et al., 2004, 2006; Eilers and Marx, 2002; Eilers et al., 2006) to study recent modifications in the distribution of ages at death in the various low mortality countries selected. Thus, successive calendar years are no longer treated independently because the additional dimension in this model enables us to take into account the continuous nature of mortality progression along ages and over the years simultaneously. In comparison to other methods for modelling mortality over age and over time, such as the Brass method (Brass, 1971), the age-period-cohort model (Clayton and Schifflers, 1987) and Lee-Carter approaches (Lee and Carter, 1992; Brouhns et al., 2002), the method of P-splines does not make any rigid assumption about the functional form of the mortality surface (Camarda, 2008). This leads to a finer expression of the underlying mortality patterns over age and time, as described by the data.

In a nutshell, the method of P-splines is a nonparametric approach which combines the concepts of B-spline and penalized likelihood. The idea behind the method is that B-splines provide flexibility which leads to an accurate fit of the data, while the penalty, which acts on the coefficients of adjacent B-splines, ensures that the resulting fit behaves smoothly. For those unfamiliar with B-splines, the term *B-spline* is short for basis spline. As splines in general, B-plines are made out of polynomial pieces that are joined together at points called *knots*. The degree of the B-splines (eg. 1: linear, 2: quadratic, 3: cubic, etc.) is given by the degree of the polynomial pieces used to build them. We used cubic B-splines. What makes B-splines attractive is that each B-spline is non-zero on a limited range of the interval over which the smoothing procedure is taking place. This also means that in any given point of the interval, only a few B-splines are non-zero, thus offering great *local* control in the resulting fit. Increasing the number of knots in the interval will increase the amount of B-splines and enhance the ability to capture variation in the data. With the method of P-splines, knots are equally spaced over the entire interval and we use a relatively large number of them, knowing that the penalty will prevent overfitting of the data by ensuring a smooth fit.

In the next two subsections, we provide an overview of the P-splines smoothing method in the context of mortality data. We start by describing how the procedure works when the aim is to perform one-dimensional smoothing over ages, as in Ouellette and Bourbeau (2009). Then, we present the method of P-splines in two dimensions which is used in this paper to smooth mortality data over ages and years simultaneously.

Smoothing in one dimension

Let d_i and e_i denote respectively the observed deaths counts and exposure data by age i for a given year in the population under study. Also, let μ_i be the force of mortality at age i. Assuming that the force of mortality is a piecewise constant function, meaning that it is constant within each single age interval such as $\mu(x) = \mu_i$ for all $x \in [i, i+1)$, then $d_i \sim \text{Poisson}(e_i \cdot \mu_i)$. Thus, in order to estimate μ_i, we use a Poisson regression model such that

$$\begin{aligned} \ln(\mathrm{E}[\boldsymbol{d}]) &= \ln(\boldsymbol{e} \cdot \boldsymbol{\mu}) \\ &= \ln(\boldsymbol{e}) + \ln(\boldsymbol{\mu}), \end{aligned}$$

where $\boldsymbol{d}$, $\boldsymbol{e}$, and $\boldsymbol{\mu}$ respectively correspond to observed deaths, exposure, and force of mortality vectors (each one including all the age-specific information). A smooth estimate of $\boldsymbol{\mu}$ is obtained by the method of P-splines (Eilers and Marx, 1996):

$$\ln(\hat{\boldsymbol{\mu}}) = \boldsymbol{B}\hat{\boldsymbol{\alpha}},$$

where $\boldsymbol{B}$ is the B-spline basis regression matrix and $\hat{\boldsymbol{\alpha}}$ is the vector of estimated coefficients associated to each B-spline included in the basis. The vector of coefficients $\boldsymbol{\alpha}$ is estimated according to a maximum likelihood procedure where the penalized log-likelihood function to maximize corresponds to

$$l^* = l(\boldsymbol{\alpha}; \boldsymbol{B}; \boldsymbol{d}) - \frac{1}{2}\boldsymbol{\alpha}'\boldsymbol{P}\boldsymbol{\alpha}. \tag{1}$$

The first term on the right-hand side of this equation corresponds to the usual log-likelihood function for a generalized linear model. The second term is a penalty term ($\boldsymbol{P}$ is a penalty matrix), which forces the estimated coefficients of adjacent B-splines to vary smoothly. The trade-off between smoothness and fidelity of the model to the observed data is tuned by a smoothing parameter, which is selected according to the Bayesian Information Criterion (see Currie et al. (2004) and Camarda (2008) for further details on the penalty term and on the smoothing parameter in the context of mortality data).

Smoothing in two dimensions

To move from one- to two-dimensional Poisson P-spline smoothing, a new B-spline basis adapted for two-dimensional regression is required. Let $\boldsymbol{B}_a$ denote the B-spline basis regression matrix for ages. Similarly, let $\boldsymbol{B}_y$ be the B-spline basis regression matrix for calendar years. The new regression matrix $\boldsymbol{B}$ to be used for two-dimesional Poisson P-spline smoothing is the following:

$$\boldsymbol{B} = \boldsymbol{B}_y \otimes \boldsymbol{B}_a, \tag{2}$$

where the symbol $\otimes$ represents the Kronecker product (Horn and Johnson, 1991, Chap. 4).

As shown in Figure 1, the Kronecker product of two B-splines (one along the year dimension and one along the age dimension) gives rise to an hill. Thus, the Kronecker product of the two B-spline basis $\boldsymbol{B}_y$ and $\boldsymbol{B}_a$ in equation (2) will populate the age-year grid with several overlapping and regularly spaced hills such as the one of Figure 1. Indeed, the complete illustration of $\boldsymbol{B}$ (not shown here) includes about 300 overlapping hills and provides great flexibility in the modelling process.

Let matrices $\boldsymbol{D}$ and $\boldsymbol{E}$ denote respectively deaths and exposure data for the population under study, where lines refer to ages and columns to calendar years. In other words, matrices $\boldsymbol{D}$ and $\boldsymbol{E}$ both include as many lines as there are ages considered and as many columns as there are calendar years considered. For the purpose of regression, these data are arranged into column vectors $\boldsymbol{d} = \text{vec}(\boldsymbol{D})$ and $\boldsymbol{e} = \text{vec}(\boldsymbol{E})$; the *vec* operator vectorizes a given matrix by

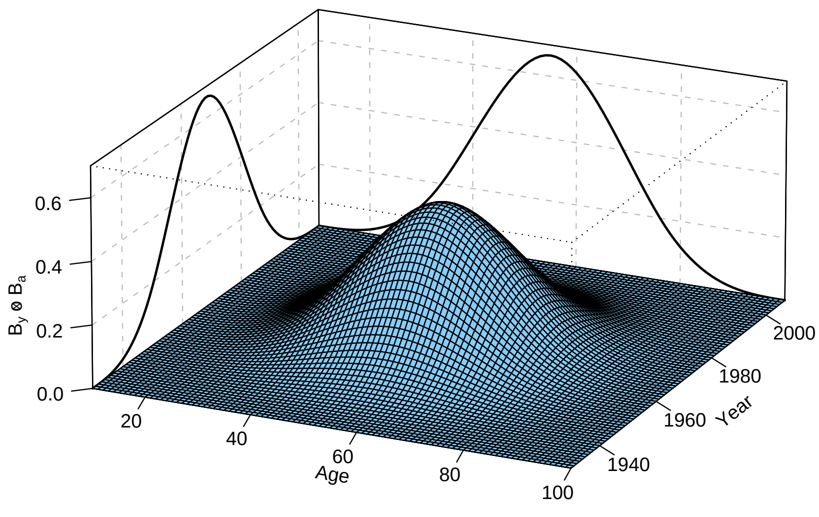


Figure 1: Two-dimensional Kronecker product of two cubic B-splines

stacking its columns. Also, let $\boldsymbol{\mu}$ be the force of mortality, similarly arranged into a column vector.

Following the same idea as in the one-dimensional case described above, we assume that the force of mortality is constant within each single age-year interval. Thus, the Poisson regression setting, together with the method of P-splines yield

$$\begin{aligned} \ln(\hat{\mathrm{E}}[\boldsymbol{d}]) &= \ln(\boldsymbol{e}) + \ln(\hat{\boldsymbol{\mu}}) \\ &= \ln(\boldsymbol{e}) + \boldsymbol{B}\hat{\boldsymbol{\alpha}}. \end{aligned} \tag{3}$$

The vector of coefficients $\boldsymbol{\alpha}$ is estimated by maximizing the penalized log-likelihood function given by equation (1), where $\boldsymbol{B}$ is defined as in equation (2) and the penalty matrix $\boldsymbol{P}$ ensures that neighbouring estimated coefficients vary smoothly over the age and year dimensions (see Currie et al. (2004) and Camarda (2008) for further details on the penalty matrix). The trade-off between smoothness and accuracy of the estimated model is controlled by two smoothing parameters, one in each dimension. These smoothing parameters are selected independently according to the Bayesian Information Criterion (see Currie et al. (2004) and Camarda (2008) for further details on the selection of the smoothing parameters in the context of mortality data), thus allowing a different amount of smoothing in each dimension.

More explicitly, from equations (2) and (3), the smoothed force of mortality corresponds to:

$$\begin{aligned}\hat{\boldsymbol{\mu}} &= \exp(\boldsymbol{B}\hat{\boldsymbol{\alpha}}) \\ &= \exp((\boldsymbol{B}_y \otimes \boldsymbol{B}_a)\hat{\boldsymbol{\alpha}}).\end{aligned}$$

For example, Figure 2 shows observed and smoothed death rates among Danish males from 1950 to 2007. Another informative way to display death rates by age and year is on shaded contour maps (Vaupel et al., 1997) often called *mortality surfaces*. Such mortality surfaces summarize a great amount of information on a single graphic representation. They provide interpretation at a glance and are very useful in the context of comparisons between countries. Smoothed mortality surfaces by sex for all countries under study are presented in Appendix. The next subsection explains how smoothed two-dimensional age-at-death distributions are computed from smoothed forces of mortality and employed in this paper.

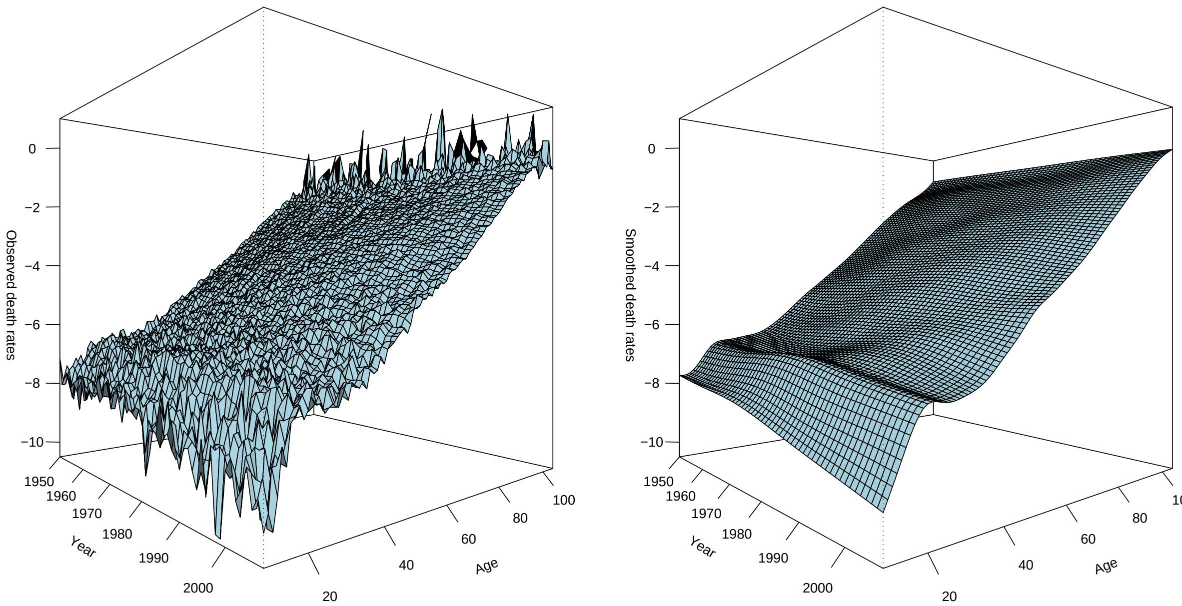


Figure 2: Observed death rates (left) and smoothed death rates using 2D Poisson P-spline smoothing (right), Danish males, 1950 to 2007. *Source*: HMD (2010)

Computation and use of smoothed age-at-death distributions

The force of mortality, the survival function and the density function are three particularly useful functions for describing mortality distributions. They also share the following interesting property: if one of these three functions is known, the remaining two can be uniquely determined. Thus, from sex-specific smoothed forces of mortality for each province, we can compute their corresponding smoothed density functions describing the age-at-death distributions in these populations.

Let $\mu(a, y)$ denote the force of mortality, a continuous function of age a and time y in a given population. Similarly, let $S(a, y)$ and $f(a, y)$ be respectively the survival function and density function, which are also continuous functions of age and time. Given the usual correspondence between these three functions (Klein and Moeschberger, 1997, Chap. 2), we have

$$\begin{aligned} f(a, y) &= \mu(a, y)\ S(a, y) \\ &= \mu(a, y)\ \exp\left(-\int_0^a \mu(u, y)\ du\right). \end{aligned} \tag{4}$$

Therefore, if the smoothed force of mortality $\hat{\mu}$ is known, then the corresponding smoothed density function $\hat{f}$ describing the two-dimensional age-at-death distribution can be computed using equation (4) and standard numerical integration techniques. For example, Figure 3 presents the smoothed density function for Danish males between 1950 and 2007.

In order to monitor changes in the central tendency and old-age dispersion of adult deaths over time and across provinces, we used the following summary measures respectively: the modal age at death and the standard deviation of individual life durations above the modal age at death. Inspired by Lexis's concept of normal life durations (Lexis, 1877, 1878), Kannisto (2000, 2001) suggested this set of measures to focus specifically on changes in the age-at-death distribution occurring at older ages. This approach, with emphasis on the modal age at death, has drawn much attention and was elaborated further in several recent studies (Cheung et al., 2005, 2008, 2009; Cheung and Robine, 2007; Canudas-Romo, 2008; Ouellette and Bourbeau, 2009; Thatcher et al., 2010).

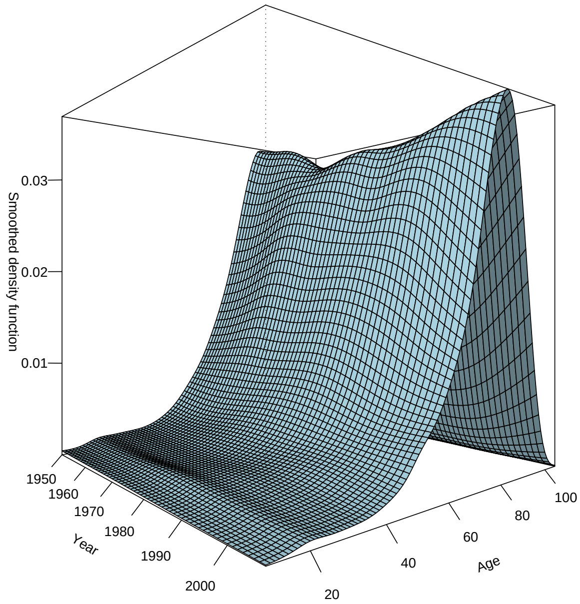


Figure 3: Two-dimensional density function describing the age-at-death distribution, Danish males, 1950 to 2007. *Source*: HMD (2010)

From the smoothed density function $\hat{f}$ describing the two-dimensional age-at-death distribution, we first computed the modal age at death using

$$\hat{M}(y) = \max_a \hat{f}(a, y),$$

and then the standard deviation of individual life durations above the modal age at death (referred to hereafter as the standard deviation above the mode) using

$$\widehat{SD(M+)}(y) = \sqrt{\frac{\int_{\hat{M}(y)}^{\infty} (a - \hat{M}(y))^2 \, \hat{f}(a, y) \, da}{\int_{\hat{M}(y)}^{\infty} \hat{f}(a, y) \, da}}.$$

Since $\hat{f}$ is a function of ages and years, $\hat{M}$ and $\widehat{SD(M+)}$ are functions of years.

RESULTS

Figure 4 presents female changes in the estimated modal age at death $\hat{M}$ and standard deviation above the mode $\widehat{SD(M+)}$ in Canada, the U.S. and the other low mortality countries under study. From the top panel of the figure, we see that females in most countries showed a sustained upward linear trend for $\hat{M}$ throughout the period under study. The average rate of increase for Japanese females has clearly been the highest of all, especially between 1960 and 2002 where it reached 3.3 months per year. However, an unexpected slowdown in $\hat{M}$ increasing trend occurred afterwards. The declining trend for US females from 2001 and onward was also largely unexpected. Consequently, the modal age at death for US females was estimated at 86.7 years in 2006, a level comparable to what they had achieved more than a decade earlier. Among all ten countries analysed, US females were the only ones to record such an obvious decrease in $\hat{M}$ for recent years. In Denmark and the Netherlands, increases in $\hat{M}$ since 1950 have been less steady than in most of the other countries (except the U.S.). Indeed, long pauses were recorded during the 1980s, 1990s, and even more recently among Danish females. In Canada, the female modal age at death has always been quite high since 1950 compared to the other countries and the upward trend continues in the latest years.

Based on the bottom panel of Figure 4, we find that females of all countries displayed lower $\widehat{SD(M+)}$ at the end of the period studied compared to 1950. Thus, old-age compression of mortality has occurred since the beginning of the second half of the twentieth century among these females. However, for most countries, $\widehat{SD(M+)}$ did not decline steadily over the entire period. For example, $\widehat{SD(M+)}$ stagnated at about 7 years from 1950 up to the early 1970s among Swedish females. Furthermore, in the nine countries excluding Japan, the 1960s were years of very slow decline. Among Japanese females, the fall in $\widehat{SD(M+)}$ has levelled off since the early 1990s. Given that their $\hat{M}$ rose rapidly during most of that period, their distribution of ages at death at very old ages has been shifting to the right without changing its shape for several years now, thus providing support to the shifting mortality scenario. Females from some of the other countries have also been thru such shifting episodes between 1950 and 2006 or 2007, but none has experienced one for as many successive years as the Japanese's. The upward trend in $\widehat{SD(M+)}$ for US females in the last few years under study

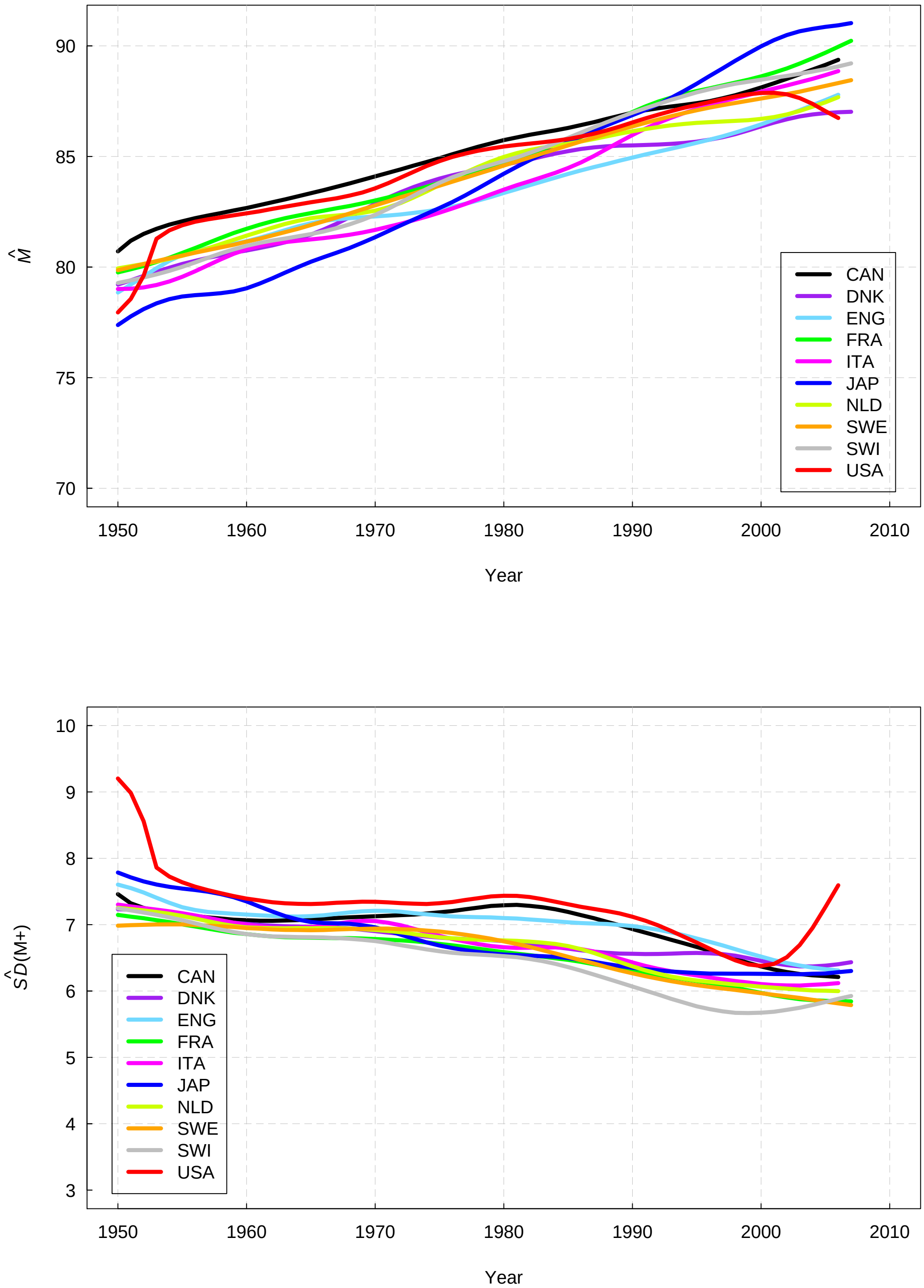
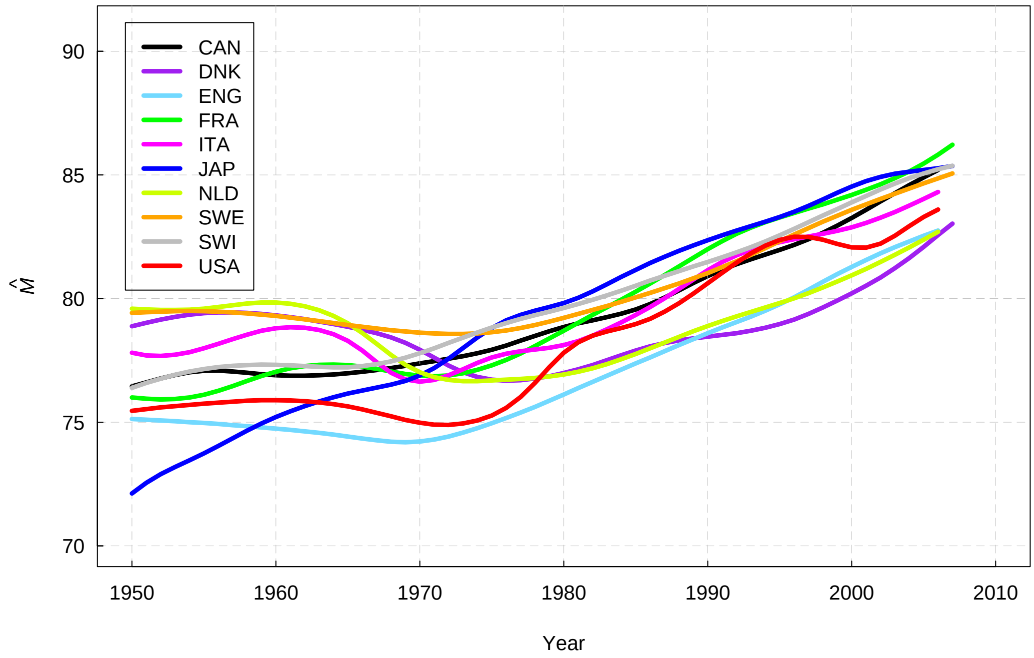


Figure 4: Estimated modal age at death (top) and standard deviation above it (bottom) using 2D Poisson P-spline smoothing since 1950, females. *Source*: HMD (2010)

stands out. It is most probably related to the recent unexpected fall in $\hat{M}$ discussed earlier.

Figure 5 shows male country-specific changes in the estimated modal age at death $\hat{M}$ and standard deviation above the mode $\widehat{SD(M+)}$. The top panel reveals that upward linear trends for $\hat{M}$ did not start much before the 1970s among males, except in Japan. Indeed, up to the 1970s, $\hat{M}$ rather stagnated or even decreased, probably because mortality reductions at ages older than $\hat{M}$ essential for its increase (Canudas-Romo, 2010) were very limited during those years. Afterwards however, the average rate of increase for males of all countries taken together was around 2.5 months per year and thus higher than among all females except those of Japan. The case of Japanese males stands out since $\hat{M}$ increased almost systematically over the whole period. Nevertheless, the rate of increase slowed down substantially among Japanese males during recent years. Thus, $\hat{M}$ for French males has been above that of Japanese males since 2005 and Swiss and Canadian males have now caught up with the Japanese's. In the U.S., the upward trend since the mid-1970s for $\hat{M}$ has not been as steady as in the other countries. Indeed, $\hat{M}$ decreased abruptly in the late 1990s, but the increasing trend has resumed in the latest years.

If we focus on the bottom panel of Figure 5, we see that $\widehat{SD(M+)}$ was not necessarily lower at the end of the period under study than in 1950. In other words, unlike what was found among females, compression above the mode did not occur in all ten countries over the period as a whole for males. Indeed, in Denmark, Italy and the Netherlands, $\widehat{SD(M+)}$ was around 7 years in both 1950 and 2006 or 2007. In between those years, $\widehat{SD(M+)}$ rose and then fell, thus revealing successive episodes of old-age decompression and compression of mortality. However, if we limit ourselves to the period starting with the onset of $\hat{M}$ increase for each country (around the 1970s, except Japan), it coincides with episodes of old-age compression of mortality because $\widehat{SD(M+)}$ was falling. Once again, the male situation in the U.S. for recent years differs substantially from other countries. Indeed, during the end of the 1990s and the beginning of the 1980s, $\hat{M}$ was falling and $\widehat{SD(M+)}$ was rising. Then, as increases in $\hat{M}$ resumed, $\widehat{SD(M+)}$ stopped increasing and remained more or less at the same high level ever since. The case of Japanese males is also worth noting because $\widehat{SD(M+)}$ has been stagnating since the early 1990s and this has been paralleled by increases in $\hat{M}$. Thus, their



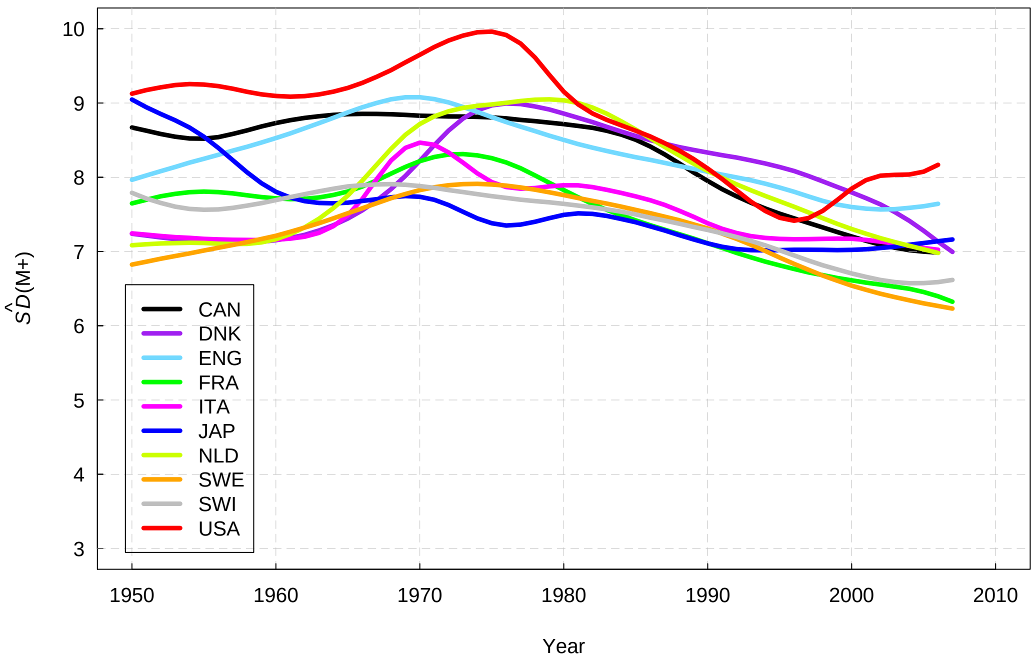


Figure 5: Estimated modal age at death (top) and standard deviation above it (bottom) using 2D Poisson P-spline smoothing since 1950, males. *Source*: HMD (2010)

distribution of ages at death at very old ages has shifted to the right, keeping an intact shape. Support to the shifting mortality scenario among males is not clear in any other country yet.

SUMMARY AND DISCUSSION

The two-dimensional smoothing method presented in this paper is a generalization of previous work by Ouellette and Bourbeau (2009) and it does not make any rigid assumption about the functional form of mortality rates along ages and calendar years. It solely rests on the assumption that mortality changes over ages and years are regular, and that the erratic behaviour is essentially explained by the randomness of rates (see figure 2). For countries with reliable data such as those included in the present work, this flexible smoothing approach represents a natural choice, notably for monitoring changes that have occurred at older ages over time.

The modal age at death and the standard deviation of individual life durations above the mode were used to summarize changes in the age-at-death distribution at older ages since 1950 in Canada, the U.S., and eight additional low mortality countries. The results show that the modal age at death generally followed an upward steady trend over this entire period among females and ever since the 1970s among males. While these increases were recorded, decreases in the standard deviation of deaths above the mode usually occurred.

However, in the last fifteen years, deviations from such regime of old-age mortality compression were observed. Indeed, since the 1990s, Japanese females and males showed convincing evidence of the shifting mortality scenario. In other words, their age-at-death distribution at older ages has been sliding towards advanced ages keeping an exact shape for over a decade now, thus increasing the likelihood that individuals will reach very old ages.

Also noteworthy is the experience of the U.S. because among all ten low mortality countries studied in this paper, the U.S. revealed the most worrying picture for the latest two decades. Indeed, for several consecutive years in that timeframe, US females and males have both recorded sharp declines in their modal age at death. These findings were unexpected and call for further investigation, as they suggest that mortality conditions at ages surrounding the modal age at death either deteriorated or did not improve uniformly during those years. Furthermore, since 1950, US females and males almost systematically exhibited high levels of variability of age at death above the mode compared to the other countries. Towards the end of the study period, they were clearly displaying the highest levels of dispersion of death in old-age.

The explanation for these results is probably not simple, but the large body of literature documenting socioeconomic inequalities prevailing in the U.S. and major differentials with respect to health plan coverage, health care access, and health care utilization (Murray et al., 2006) could account for some part of it. An analysis of changes in the age-at-death distribution at older ages by socioeconomic group or by region could improve our current understanding of the recent mortality dynamics recorded among US adults. Since the method of P-splines in two dimensions takes simultaneously into account the continuous nature of mortality progression over ages and time, it is particularly well-suited to the study of countries with small population size or for analysing population subgroups. It could therefore be particularly useful in these analyses.

Given that the HMD data for the U.S. rests upon official publications from the US Census bureau and the National Center for Health Statistics, another interesting avenue to reinforce these results would consist in replicating the analysis presented in this paper using on another source of mortality data at older ages. The experience of a subset of Medicare Social Security Administration records could be helpful in that sense.

In conclusion, although Canada and the U.S. are neighbouring countries, our results for the former regarding recent old-age mortality trends, especially over the last two decades, are closer to those obtained for the remaining eight low mortality countries studied. Canada's publicly funded health care system which offers universal coverage and provides medical health care services on the basis of need rather than on the ability to pay could play a role in the multifaceted explanation for this finding.

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Appendix

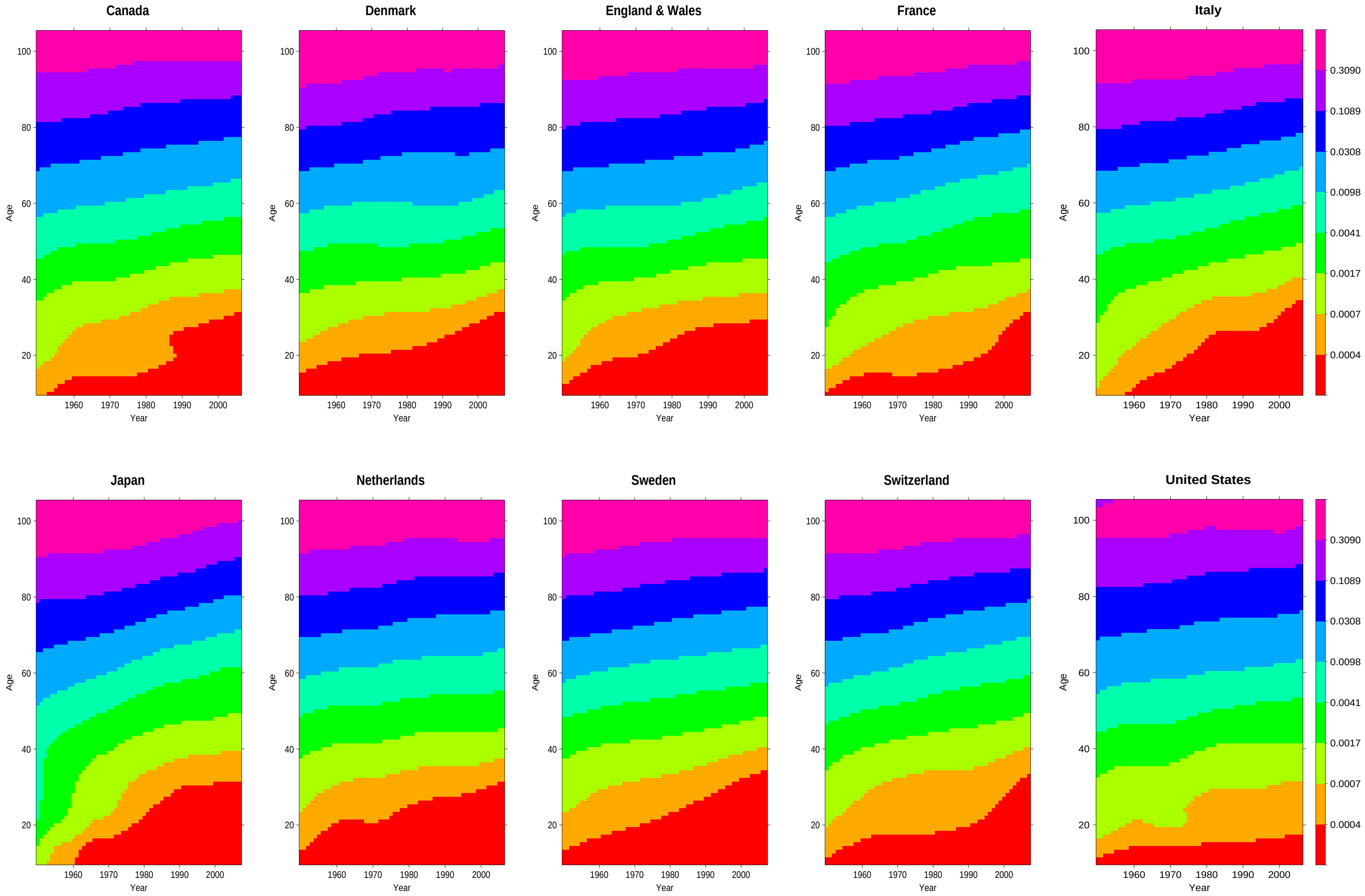

Figure 6: Country-specific smoothed mortality surfaces among females since 1950. *Source*: HMD (2010)

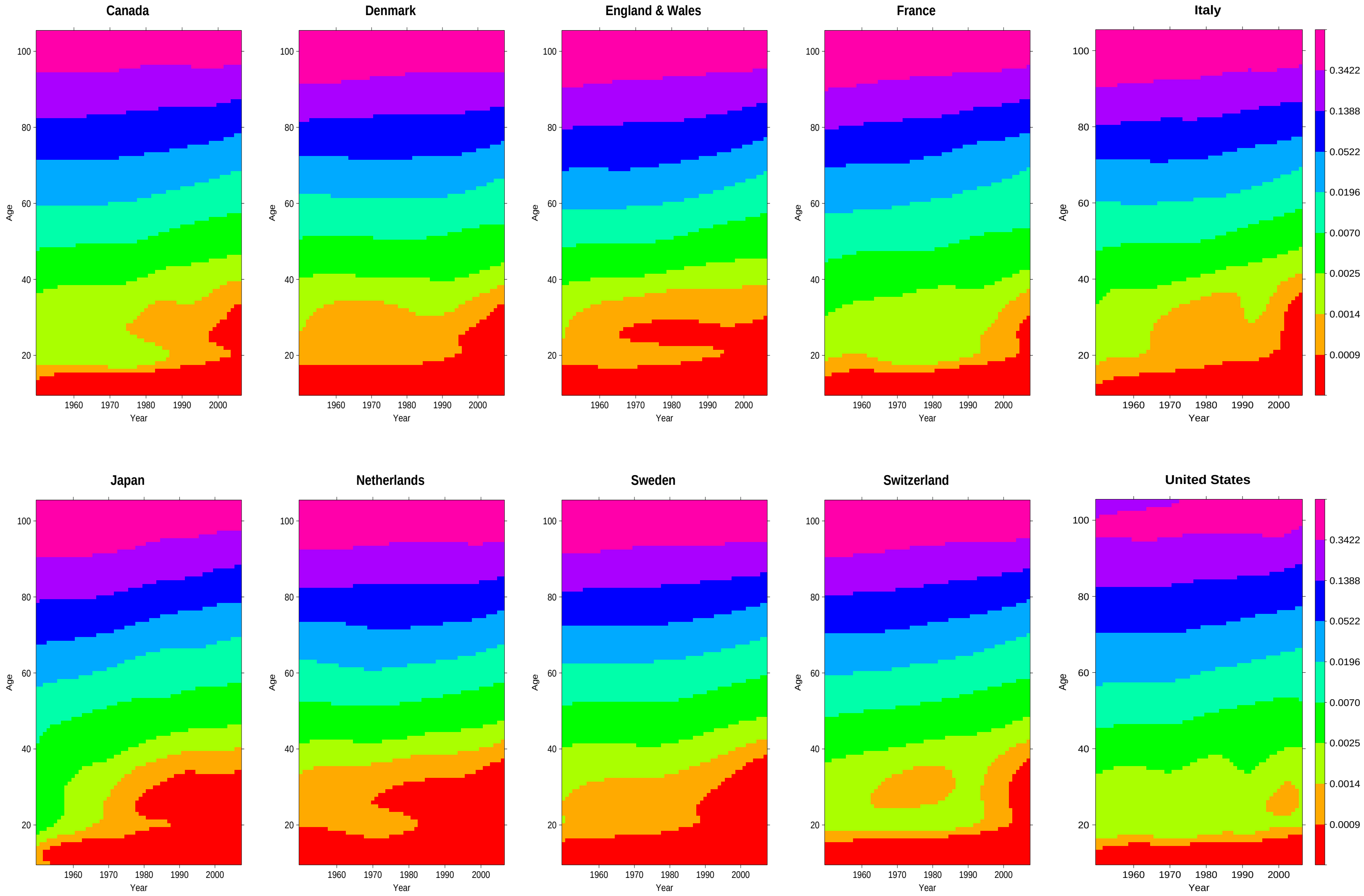

Figure 7: Country-specific smoothed mortality surfaces among males since 1950. *Source*: HMD (2010)