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Modeling and Forecasting Chinese Population Dynamics in a Multi-Population Context

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Modeling and Forecasting Chinese Population Dynamics in a Multi-Population Context

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Executive Summary

This report documents the “China Research Topic” project commissioned by the Society of Actuaries. The purpose of this project is to forecast China’s population structure in the coming decades by projecting both the mortality and fertility rates of the Chinese population. In particular, we forecast China’s future mortality rates in a multiple-population context, by explicitly allowing its systematic mortality patterns to gradually converge to those of a group of more developed countries with higher life expectancy levels and better data quality. The implications of population structure to China’s social security system will also be discussed.

The deliverables of this project include the following:

1. A discussion of important issues regarding China’s mortality and fertility data.

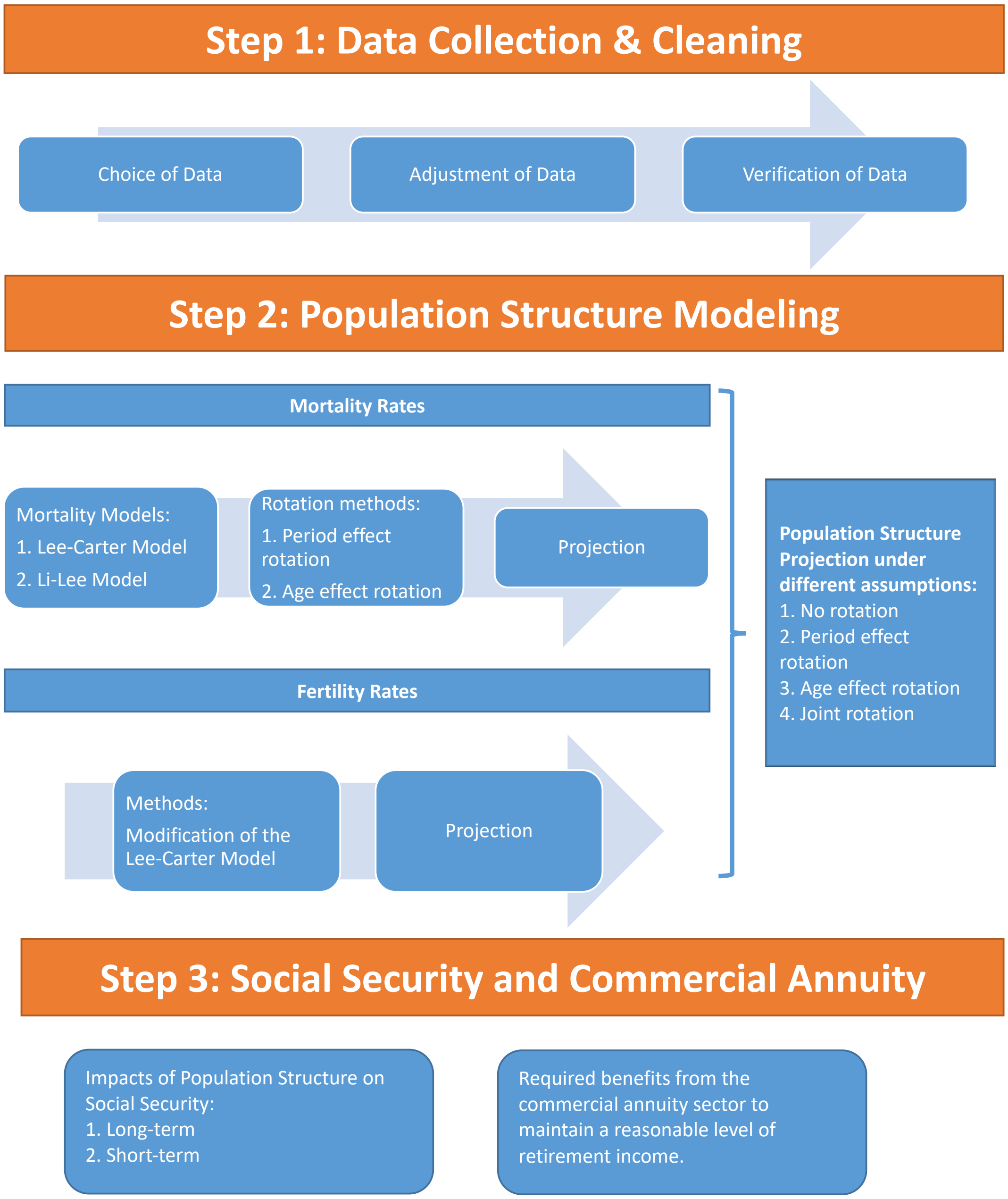
2. A summary of the mortality modeling methods used in this project.

3. Detailed analysis of China’s demographic projection with (a) mortality projections under the assumption of convergence in systematic mortality pattern towards a group of more developed countries, and (b) projections of age-specific fertility rates.

4. Assessment of the impact of population structure on China’s social security system.

The procedure of the project is summarized in Figure 1. The report is structured as follows. Section 1 discusses the motivation for implementing the mortality rotation method on China’s mortality patterns towards those of a group of more developed countries. Section 2 defines the group of develop countries used as the benchmark, and describes the data sets of mortality rates of China and the developed countries, as well as the fertility rates of China. Section 3 defines the notation and summarize the mortality models used in this project. Section 4 implements these mortality models on historical mortality data, and analyze the key mortality patterns of China and other countries. Section 5 introduces the algorithm allowing for convergence of key mortality patterns of China towards those of the developed countries. This algorithm is referred to as "mortality rotation" in this project. We also assess the impact of the mortality rotation algorithm on forecasts of China's mortality. Section 6 describes the algorithm used to forecast China's age-specific fertility rates and the forecasting results. Section 7 combines mortality and fertility forecasts to project China's population structure in different horizons. Section 8 discusses the implication of population structure on China's social security system. Section 9 discusses the implication of the mortality rotation assumptions on the commercial annuity sector in China. Section 10 concludes.

Figure 1: Flow chart of this project.



Contents

1 Motivations for Mortality Rotation

Life expectancy in China, along with its economic level, has grown substantially over the past decades. According to the statistics from the World Bank, China's life expectancy at birth had increased by around 75%, i.e., from 43.4 to 76 years, from 1960 to 2015.[1] In particular, this increase rate has been faster than in many other developed countries such as the US. Moreover, the increase in life expectancy in China during this period has been largely due to mortality improvement of the working age, rather than the older ages, while the latter ages have been documented as the major driver of mortality improvement in more developed countries in the second half of the 20th century (Li et al., 2013).

In fact, it remains an open question as to whether the historical patterns in China's mortality improvement will continue in the future. First, existing studies suggest that countries with a higher life expectancy level tend to experience slower mortality improvements than those with a middle life expectancy level (Raftery et al., 2013). Given the current life expectancy at birth, it is likely that the increase rate of China's life expectancy level will slow down in the future. Moreover, mortality reductions for the Chinese working age population were largely due to improvements of the national health care system during the second half of the 20th century, and especially to the adoption of advanced treatments for infectious diseases, such as malaria and tuberculosis (World Health Organization, 1999). Today, mortality rates for the Chinese working age population are already low, and it is questionable whether there will be other factors leading to similar degree of mortality declines in the next 5 or 6 decades. Meanwhile, mortality decline for population over age 60 had been slow, but improved living and medical conditions in recent years, such as better nutrition, healthier life-style, and advancements in treatments for critical illness (e.g., heart attack) could potentially facilitate mortality declines for the elderly.

Therefore, when forecasting life expectancy of China's population, it is very important to take such potential changes in mortality patterns into account. However, as will become

[1] Source: `https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations=CN`.

clear in Section 2, China's mortality data is of lower quality and availability when compared with more developed countries. In particular, data issues make it extremely difficult, if possible at all, to infer potential changes in China's mortality patterns in a statistically reliable way.

This project proposes an innovative method to derive long-term forecasts of China's mortality in a multiple-population context. More specifically, we use historical mortality patterns in a group of developed countries, with well established health-care systems, stable socioeconomic conditions, and higher life expectancy levels as benchmarks to infer China's future mortality development. Regarding mortality projections, a popular view in the existing literature is the *coherence* assumption: mortality projections of different countries should not diverge in the long run. In other words, the key quantities, e.g., the projected age-specific mortality levels and the life expectancies, should be similar across all countries when long term forecasts are made.

When projecting China's mortality improvement, we consider two major ingredients: the period effect and the age effect. These two effects are the considered the most important factors to summarize mortality patterns in various studies (see, for example, (Lee and Carter, 1992; Cairns et al., 2006; Booth and Tickle, 2008) and the references therein). Specifically, the period effect refers to the aggregate mortality trend, i.e., the systematic mortality decline to the whole population; while age effect refers to the age-exposure to the period effect, i.e., how sensitive each age is to the aggregate mortality trend. There coherence modeling of both the period effect and the age effect has been discussed in the existing literature. For example, Li and Lee (2005) and Hyndman et al. (2013) show that the systematic mortality trend has been rather similar for a large group of developed countries since 1950; Li and Li (2017) argue that the age-exposure to the aggregate mortality decline in many countries had substantially shifted in the 20th century, i.e., mortality declines had slowed down for younger ages and accelerated for older ages; Li et al. (2013) propose a rotation mechanism to achieve coherence of the age effect.

In this project, we use the well established mortality modeling techniques, the Lee-Carter model (Lee and Carter, 1992) and the Li-Lee model (Li and Lee, 2005), to capture the period and age mortality effects for China and the developed countries. Moreover, we extend the rotation method proposed by Li et al. (2013), and combine this method with the mortality models to achieve convergence in the period and the age effects. More details of these modeling methods are discussed in Section 4 and Section 5.

In the empirical studies, we see that the rotation of China's period and age effect is likely to have two opposite impacts on China's mortality projection. First, China's longterm aggregate mortality improvement will be slower than its historical level. Second, in the long run, key drivers of mortality improvements will gradually shift from working ages to older ages, along with the convergence of the age effect. Therefore, theoretically speaking, the rotation could either increase or reduce the projected mortality rates or life expectancy for China in the long run. We will be able to draw clearer conclusions after the rotation of age effect is implemented in the next phase.

2 The Data

This report uses the uni-sex mortality data of China and a group of 15 developed countries, as well as the fertility data of China. In this section we introduce the datasets as well as the associated preliminary treatments that we conducted.

2.1 China

2.1.1 Mortality data

Two datasets for Chinese mortality are used:

1. The Population Division of the United Nations (UN),[2] which contains gender-specific numbers of death and the corresponding populations for 5-year periods (1960 - 1965, ..., 2010 - 2015) and 5-age ranges (0-4, ..., 75-79,80+).

2. The World Health Organization (WHO),[3] which contains gender-specific numbers of death and the corresponding populations for 1-year periods (2000, ..., 2015) and 5-age ranges (0, 1-4, 5-9,... , 85+).

Before we can use these two datasets in the empirical analysis, we need to prepare them in the same format. In particular, the format of these two datasets should be consistent with the datasets for the developed countries (introduced later). In this project, the following preparations are implemented:

1. Interpolate the UN dataset into 1-year periods;

2. Adjust and extrapolate the age groups of both datasets to 0-4, 5-9, ..., 95-99;

3. Merge the two datasets.

[2]Source `http://www.un.org/en/development/desa/population/`.
[3]Source: `http://apps.who.int/healthinfo/statistics/mortality/whodpms/`.

In the first step, we use the moving average technique in the the Curve Fitting Toolbox of MATLAB [4]. As an illustration of the interpolation, Figure 2 shows the raw and interpolated mortality rates for age 55. We see that the interpolated mortality is able to accurately reflect the mortality patterns in the raw data. In the second step, we apply the Kannisto-Thatcher(KT) method (Kannisto 1996, Thatcher et al. 2002), which is currently used by the Office for National Statistic (ONS) in UK, to extend the oldest age group to 95-99. Details on this method can be found in the appendix. Finally, we create an integrated dataset with China's mortality experiences in 1-year and 5-age format from 1960 to 2015. The integrated dataset combines mortality experiences from both the UN dataset (1960 to 1999) and the WHO dataset (2000 to 2015).

As an examination of the errors introduced by the data preparation , we compare the unisex life expectancy at birth (e_0) from the integrated dataset and the one generated by the World Bank from 1960 to 2015.[5] The results are gathered in Figure 3. We see that the two sets of e_0 are rather close to each other, which means that our data preparation approach does not introduce significant biases to the data.

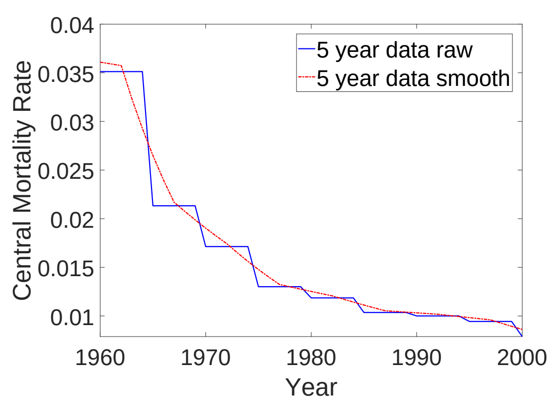


Figure 2: The raw and interpolated mortality rates for age 55 in China from the UN dataset.

[4]https://www.mathworks.com/products/curvefitting.html
[5]`https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations=CN`

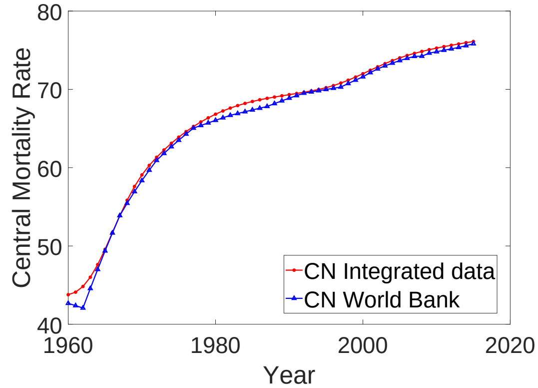


Figure 3: The unisex life expectancy at birth e_0 for China from the integrated dataset and the World Bank.

2.1.2 Fertility data

Chinese fertility data are collected in the China Population Statistics Yearbook (2003 to 2005) and the China Population and Employment Statistics Yearbook (2006 to 2015). In particular, age-specific data from age 15 to 49 are included.

2.2 The Group of More Developed Countries

We use mortality data for 15 developed countries as the benchmark: Austria, Canada, Germany, Denmark, Finland, France, Italy, The Netherlands, Norway, Spain, Switzerland, Sweden, the U.K., the U.S., and Japan. These countries are referred to as the "low-mortality countries" by Li and Lee (2005).

Mortality data for these countries are obtained from the Human Mortality Database, where death rates for age 0 to 99 and year 1956 to 2011 are available.

3 A Summary of Notation and the Mortality Methods Used

3.1 Notation

The following notation is used throughout the rest of this report:

- i represents the population index;
- x represents the age of an individual;
- t represents calendar year;
- $m_{x,t}$ is the crude death rate of age x and year t in the single population context;

Thus, $m_{i,x,t}$ is the crude death rate of population i, age x and year t in the multiple-population context.

3.2 The Lee-Carter Method

Suppose that we observe, for a single population, the crude death rates for X different ages and T years. Then the Lee-Carter model (Lee and Carter, 1992) assumes that:

$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}, \tag{1}$$

where parameter a_x measures the average level of mortality at age x; k_t is the period effect, i.e., the overall level of mortality in year t; b_x is the age effect, which measures the sensitivity of $\log m_{x,t}$ with respect to the period effect; and $\varepsilon_{x,t}$ is the error term.

For identification purpose, Lee and Carter (1992) assume that:

- $a_x = \frac{1}{t_1 - t_0 + 1}(\sum_{t=t_0}^{t_1} \log m_{x,t})$ for all x;
- $\sum_{x=x_0}^{x_1} b_x = 1$;
- $\sum_{t=t_0}^{t_1} = 0$.

The Lee-Carter model can be estimated by Singular Value Decomposition (SVD) (Lee and Carter, 1992) or maximum likelihood estimation (Wilmoth, 1993). After the parameters in (10) are obtained, mortality forecasts can be obtained by extrapolating the period effect with a time-series model. A popular choice of time-series model in the literature is the random walk with drift (also known as $ARIMA(0,1,0)$). Specifically, the dynamics of the period effect is given by

$$k_t = d + k_{t-1} + \epsilon_t, \;\; \epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2), \tag{2}$$

where d is the drift term, which needs to be estimated. For more details regarding the estimation and forecasting of the Lee-Carter model, we refer to Booth et al. (2002) .

3.3 The Li-Lee Method

The Li-Lee model (Li and Lee, 2005) is a popular multiple population model. Suppose that the dataset contains I populations, each with X ages and T years, the Li-Lee model is given by:

$$\log m_{i,x,t} = a_{i,x} + B_x K_t + b_{i,x} k_{i,t} + \varepsilon_{i,x,t}, \tag{3}$$

where $a_{i,x}$ measures the average mortality level of age x in population i; K_t is the systematic period effect, i.e., the overall mortality level in year t for all the I populations; B_x is the systematic age effect, i.e., the aggregate sensitivity of mortality at age x to the systematic period effect; $k_{i,t}$ and $b_{i,x}$ measure the mortality fluctuation around the systematic period effect for population i and the corresponding age sensitivity.

Similar to the Lee-Carter model, the Li-Lee model can be estimated by Singular Value Decomposition. After estimating the parameters in (3), mortality forecasts can be obtained by extrapolating the systematic period effect K_t and the population-specific mortality fluctuations $k_{i,t}$. Li and Lee (2005) assume a random walk with drift for the dynamics of

K_t, and a stationary $AR(1)$ process for $k_{i,t}$'s. Formally, the dynamics of K_t is given by

$$K_t = d_0 + K_{t-1} + \epsilon_t, \quad \epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2), \tag{4}$$

where d_0 is a parameter to be estimated; and the dynamics of $k_{i,t}$ is given by

$$k_{i,t} = \alpha_{0,i} + \alpha_{1,i} k_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \overset{i.i.d.}{\sim} N(0, \eta_i^2), \tag{5}$$

where $\alpha_{0,i}$, $\alpha_{1,i}$ and η_i are parameters to be estimated. From (4) and (5), we see that K_t is a non-stationary process with a persistent impact, where $k_{i,t}$'s are stationary and will converge to constants after a certain period of time. Therefore, the systematic period effect K_t determines the longterm mortality level of all the I populations.

4 Implementing the Lee-Carter method and the Li-Lee method to Mortality Data of China and the Benchmark Countries

4.1 The Lee-Carter Method on Chinese Mortality

The Lee-Carter model is applied on Chinese mortality data over 1950 to 2015 and ages 0 to 99. Figure 4 display the average mortality level a_x^{CN} age effect b_x^{CN}, and period effect k_t^{CN} for Chinese mortality.

We see that the average mortality level starts high for infant, decreases between infant to age 15, and increase afterwards. The period effect is monotonically decreasing, indicating that Chinese mortality is in general declining over the the sample period. Moreover, the decrease of the period effect is much faster in 1950 to 1975 than in the period afterwards. Finally, the age effect is monotonically decreasing as well, meaning that the age sensitivity with respect to the period effect is decreasing over age.

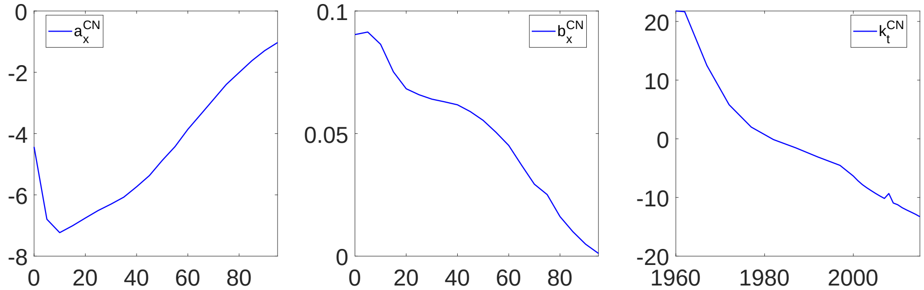


Figure 4: The estimated a_x^{CN}, b_x^{CN}, and k_t^{CN} obtained from the Lee-Carter model for China.

4.2 The Lee-Carter Method on Benchmark Mortalities

We now apply the Lee-Carter model separately on the 15 low mortality countries. The average mortality levels, age effects and period effects are shown in Figure 5, 6 and 7, respectively.

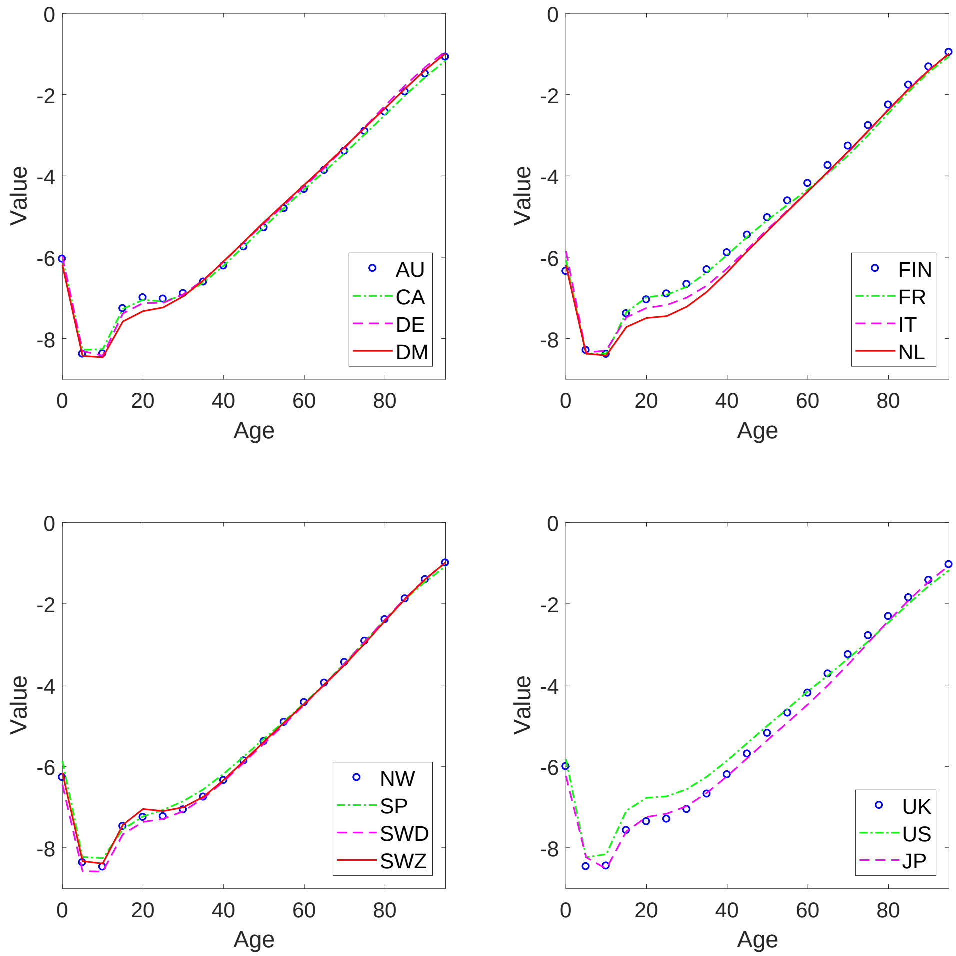


Figure 5: The estimated a_x obtained from the Lee-Carter model for 15 low mortality countries.

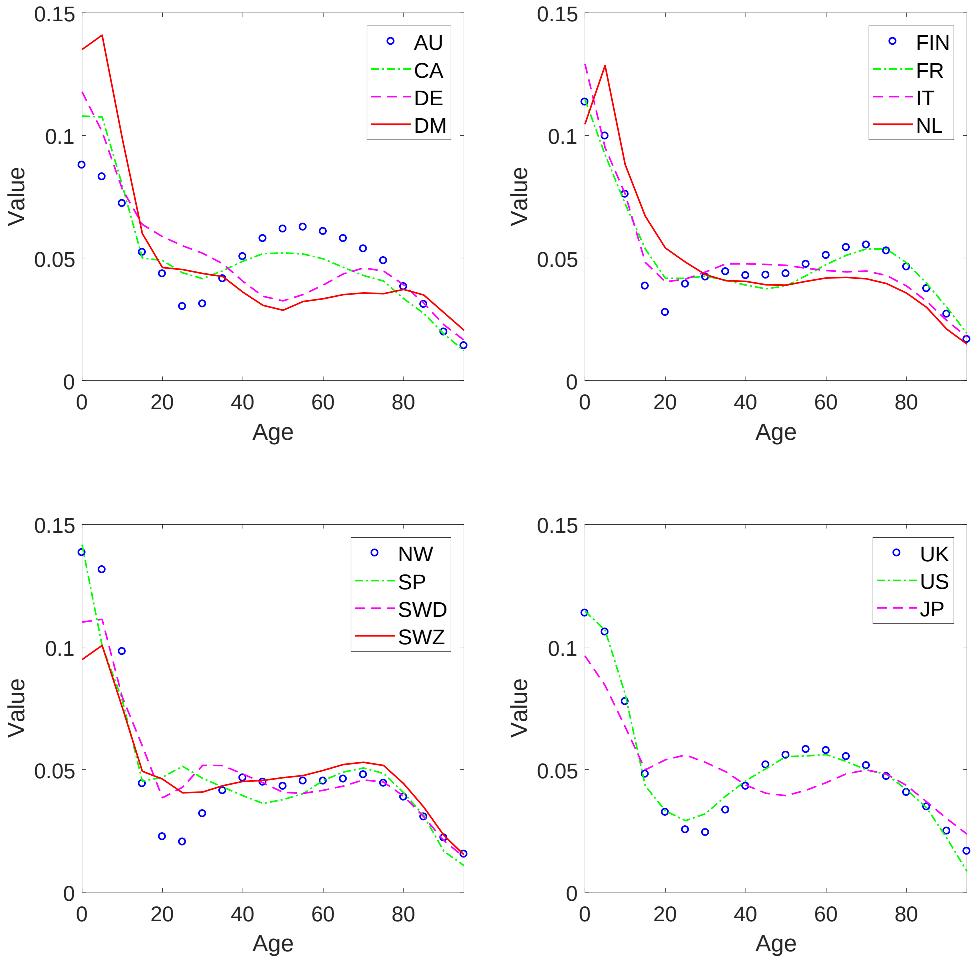


Figure 6: The estimated b_x obtained from the Lee-Carter model for 15 low mortality countries.

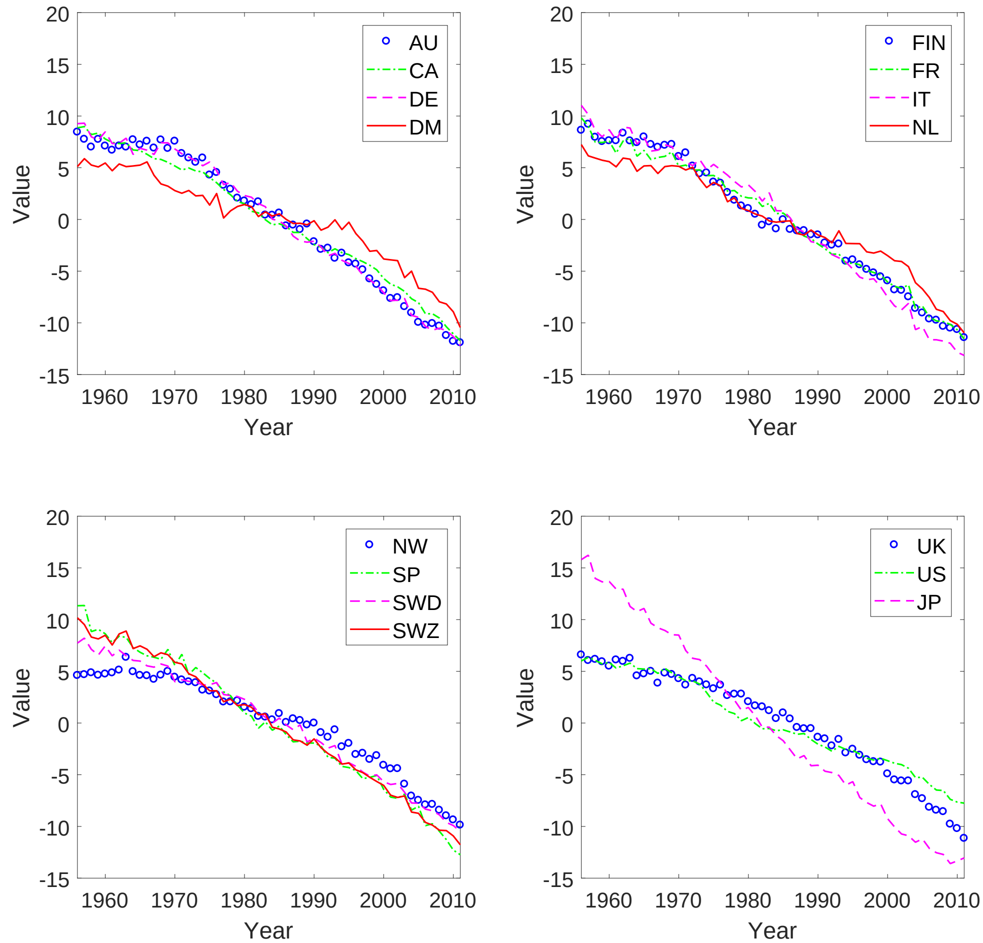


Figure 7: The estimated k_t obtained from the Lee-Carter model for 15 low mortality countries.

4.3 The Li-Lee Method on Benchmark Mortalities

Finally, we apply the Li-Lee model on the 15 low mortality countries. The average mortality level $a_{i,x}$ and the systematic period and age effect (K_t and B_x) are shown in Figure 8 to 10.

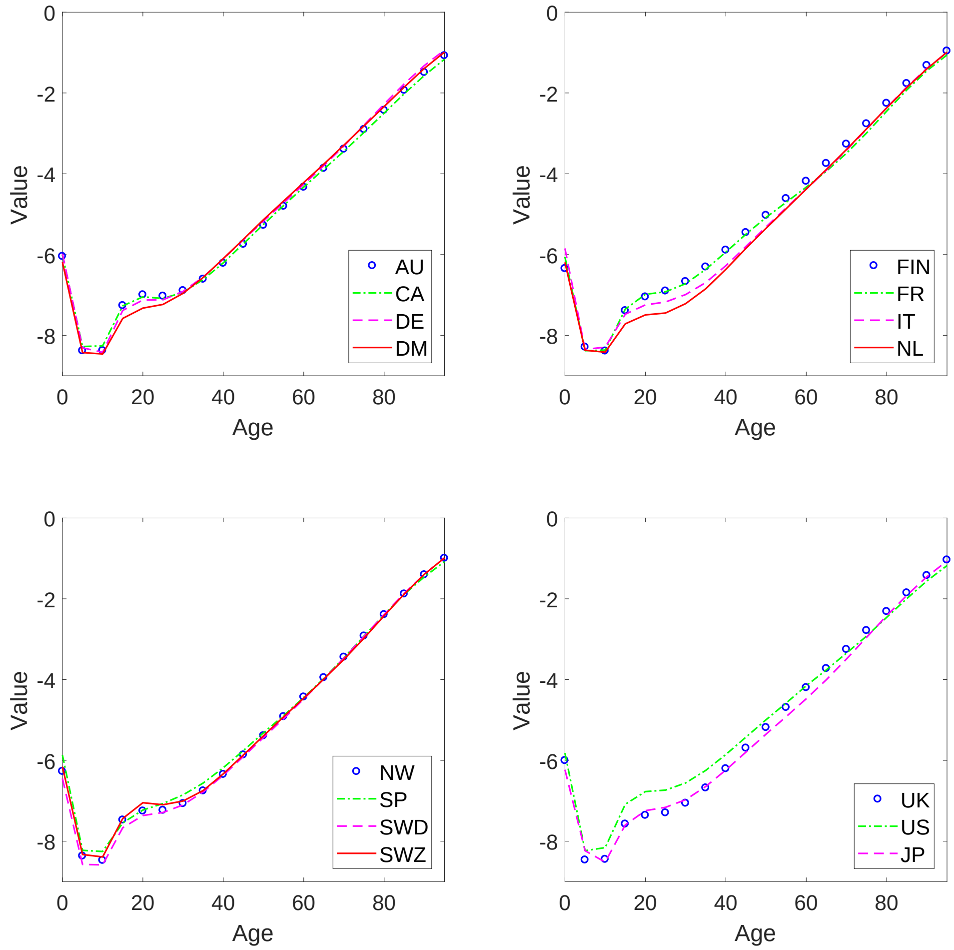


Figure 8: The estimated $a_{i,x}$ obtained from the Li-Lee model for 15 low mortality countries.

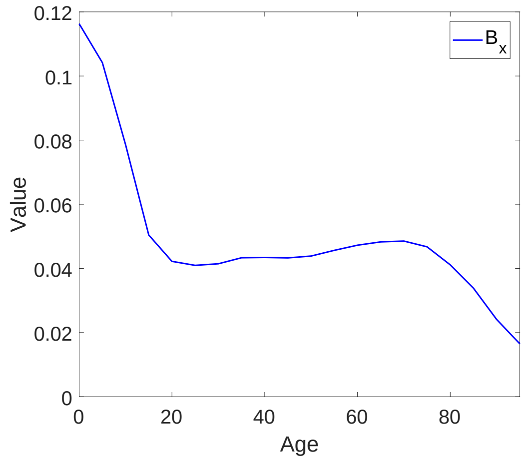


Figure 9: The estimated B_x obtained from the Li-Lee model for 15 low mortality countries.

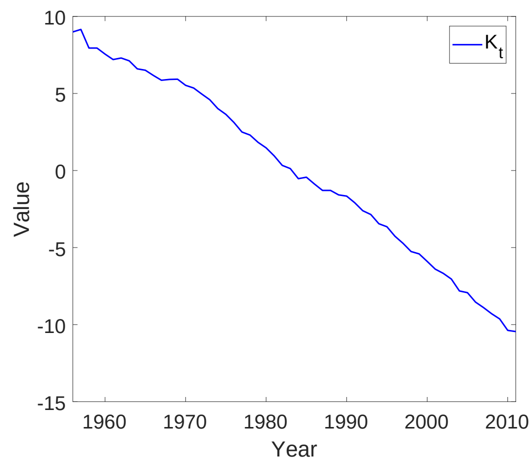


Figure 10: The estimated K_t obtained from the Li-Lee model for 15 low mortality countries.

4.4 Comparison of the Period and the Age Effect

Before making mortality projections, we first look at the historical mortality patterns for China and the 15 low mortality countries. Figure 11 to 12 display the period effect and the age effect, respectively.

Figure 11 shows that, while the period effect for the 15 mortality countries is rather close to linear, the period effect for China has different patterns: it had decreased drastically from 1950 to 1975, became more stable between 1975 and 2000, and accelerated afterwards.

Moreover, From Figure 12, we see that the age effect for China appears to be decreasing over the whole age range. More precisely, mortality improvements in China over the past 60 years concern mostly children and teenagers (age 0 to 20) and the working ages(20 to 60). In fact, China's mortality improvements during this period were largely due to improvements in the national health care system during the second half of the 20th century, and in particular to the adoption of advanced treatments for infectious diseases, such as malaria and tuberculosis ((World Health Organization, 1999)). On the other hand, the age effect of the 15 mortality countries decreases between age 0 and around 20, and increases between 20 and 75. This pattern is also observed in various existing studies (Lee and Carter, 1992; Booth et al., 2002; Li and Lee, 2005). The large age effects between age 0 and around 20 reflect the advancements of medical treatments of dreadful diseases for children and teenager; while the second hump of age effect for the middle and old ages reflect improved living and medical conditions in recent years, such as better nutrition, healthier life-style, and advancements in treatments for critical illness (e.g., heart attack).

Therefore, it seems that the age effect for China is not likely to continue in the future. First, mortality rates for the Chinese working age population are already low today, and it is questionable whether there will be other factors leading to similar degree of mortality declines in the next 5 or 6 decades. Moreover, mortality decline for population over age 60 had been slow over the same period. However, as Chinese elderly get access to better medical conditions and build up healthier lifestyles, it is reasonable to expect that their mortality experience will improve faster in the future.

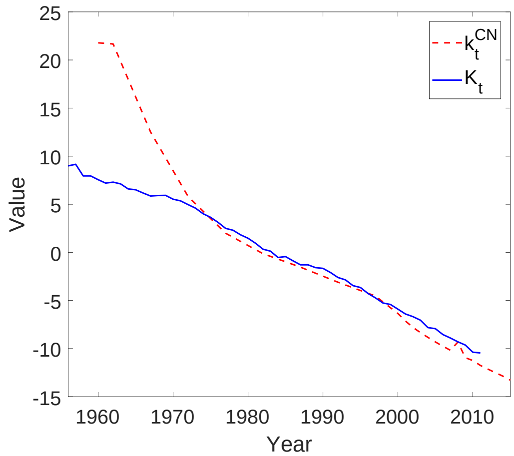


Figure 11: The estimated k_t^{CN} obtained from the Lee-Carter model for China and the estimated K_t obtained from the Li-Lee model for 15 low mortality countries.

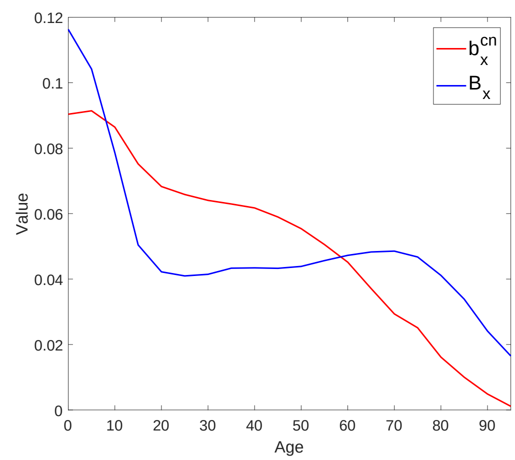


Figure 12: The estimated b_x^{CN} obtained from the Lee-Carter model for China and the estimated and projected B_x obtained from the Li-Lee model for 15 low mortality countries.

5 Improving the Lee-Carter Method for China Mortality

In this section we examine the empirical results, especially the projected life expectancy at birth for China under different assumptions of mortality rotation. In this section, Chinese mortality is fitted and projected by the Lee-Carter model, while mortality for the 15 low mortality countries are is fitted and projected by the Li-Lee model.

5.1 No Rotation

As a benchmark, we project Chinese mortality without mortality rotation. Figure 13 displays the projected (systematic) period effect for China and the 15 low mortality countries up to 2065. We see that the projected period effect for China has a steeper downward trend than that of the 15 low mortality countries, indicating a faster aggregate mortality decline for China.

Figure 14 displays the projected life expectancy at birth for the 15 mortality countries and China. We see that the increase in life expectancy at birth is faster for China than for the 15 low mortality countries, which is in line with the observation in Figure 13.

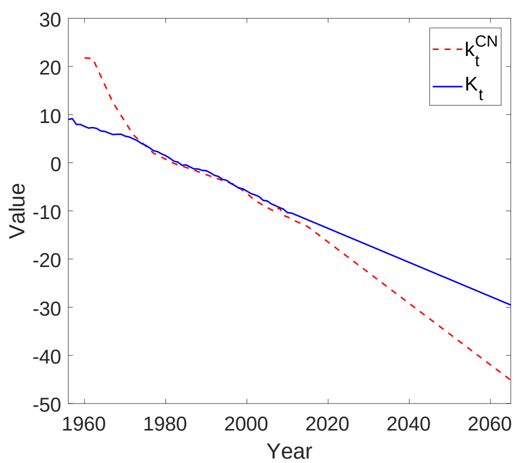


Figure 13: The estimated and projected k_t^{CN} obtained from the Lee-Carter model for China and the estimated and projected K_t obtained from the Li-Lee model for the 15 low mortality countries.

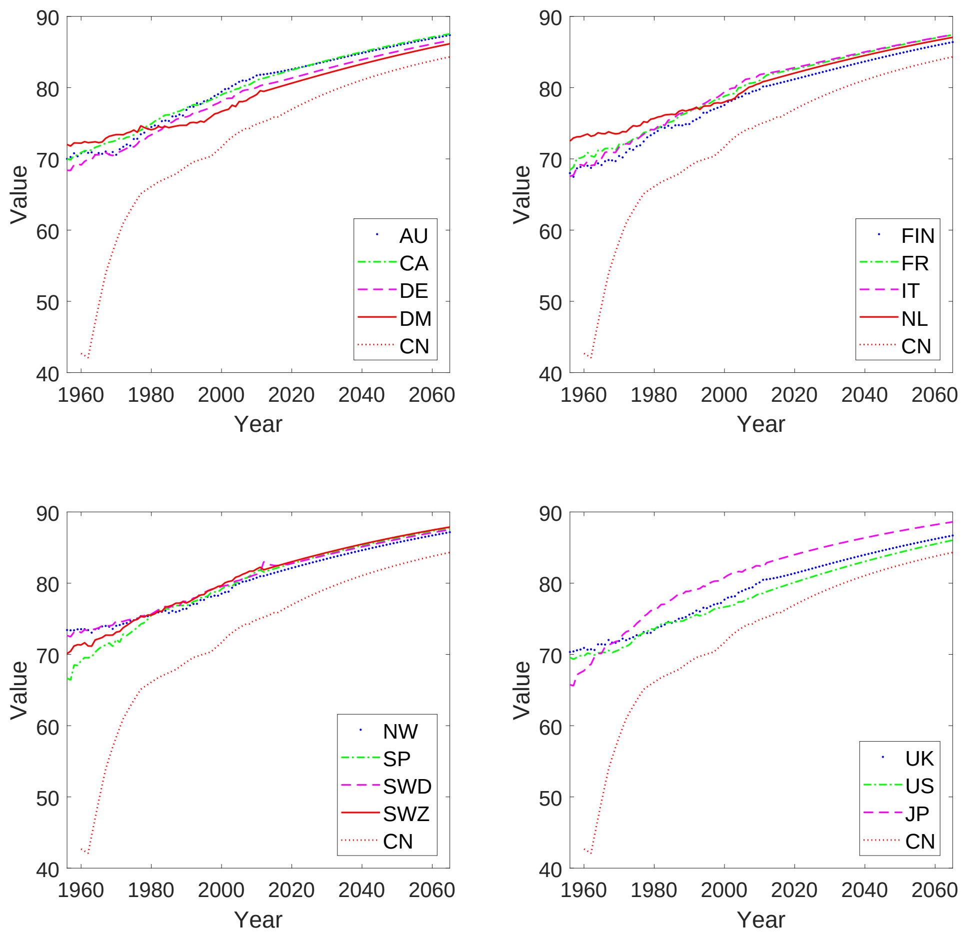


Figure 14: The estimated and projected e_0^{CN} obtained from the Lee-Carter model for China and the estimated and projected e_0 obtained from the Li-Lee model for the 15 low mortality countries.

5.2 Rotating the Period Effect

In this section, we rotate China's mortality period effect towards the systematic period effect estimated from a 15 low mortality countries. Under the rotation algorithm, China's mortality projection will depend on (a) its own historical mortality trend; and (b) the systematic mortality trend of the 15 low mortality countries. In particular, China's mortality projection will be dominated by its own historical trend in the short run. However, when the forecast horizon increases, the systematic mortality trend of the 15 low mortality countries will have a larger impact on China's mortality projection. In the long run, China's mortality will be projected to follow closely the benchmark trend instead of its own historical trend.

To implement the rotation algorithm, we first extend the Lee-Carter model by allowing the drift term of the period effect to change over time. Specifically, we have (for Chinese mortality):

$$k_t^{CN} = d_t^{CN} + k_{t-1}^{CN} + \epsilon_t^{CN}, \quad \epsilon_t^{CN} \overset{i.i.d.}{\sim} N(0, \sigma_{CN}^2) \tag{6}$$

The idea of the rotation algorithm is to let d_t^{CN} gradually rotate towards d_0, i.e., the drift term of K_t (see (4)). The algorithm is given as follows. Denote the projected Chinese period life expectancy at birth at time t by $e_0^{CN}(t)$, which is obtained by extrapolating k_t^{CN} with conditional drift term d_t^{CN} in Equation (6). We assume that the rotation starts in year 2016, i.e., the first year after the projection begins. Further, we denote by e_0^u the threshold of period life expectancy at birth where the rotation of d_t^{CN} is completed, i.e., when d_t^{CN} equals to d_0. In this way, the rotation algorithm is given below. For $t \geq 2016$, we have

$$d_t^{CN} = \begin{cases} (1 - w_t) d_o^{CN} + w_t d_0, & \text{if } e_0^{CN}(t) < e_0^u, \\ d_0, & \text{if } e_0^{CN}(t) \geq e_0^u. \end{cases} \tag{7}$$

In Equation (7), d_o^{CN} is the starting value of d_t^{CN}, which is estimated using Chinese historical mortality data, and captures the historical mortality trend in China. When the projection begins, d_t^{CN} starts to rotate towards the benchmark drift term, with a speed of rotation controlled by the time-varying weighting parameter w_t. Finally, d_t^{CN} becomes the same as the benchmark drift term when $e_0^{CN}(t)$ attains the threshold, e_0^u. In this report, we let $e_0^u = 80.99$, which is the average life expectancy at birth for the 15 low mortality countries in 2011, i.e., the end of the sample to which the Li-Lee model is estimated.

In order to complete the rotation, the weighting parameter, w_t, needs to be specified. In this report, we follow the choice by Li et al. (2013). In particular, for each $t \geq 2016$, we let

$$w_0(t) = \frac{e_0^{CN}(t) - 75.9}{e_0^u - 75.9}, \tag{8}$$

$$w(t) = \{0.5[1 + \sin[\frac{\pi}{2}(2w_0(t) - 1)]]\}. \tag{9}$$

The shape of the weighting parameter is given in Figure 15. We see that d_t^{CN} will converge to d_0 in 2048.

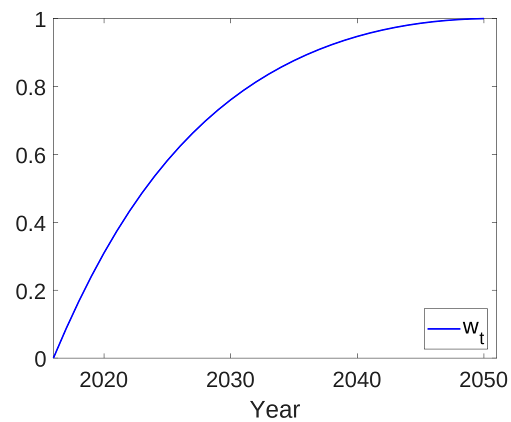


Figure 15: The rotating $w(t)$ for the drift term of period effect for China

Figure 16 shows the projected k_t^{CN} with and without rotation and the projected K_t for the 15 low mortality countries. Figure 17 shows the analogous plots of the life expectancy at birth. We see that rotating the period effect reduces both the mortality improvement trend and the speed of increase in life expectancy at birth for China.

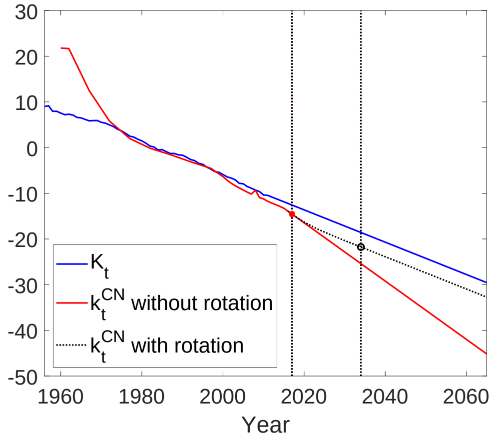


Figure 16: The estimated and projected (with/without rotation) k_t^{CN} obtained from the Lee-Carter model for China and the estimated and projected K_t obtained from the Li-Lee model for 15 low mortality countries. Two dash lines indicate the rotation points of the period effect of China

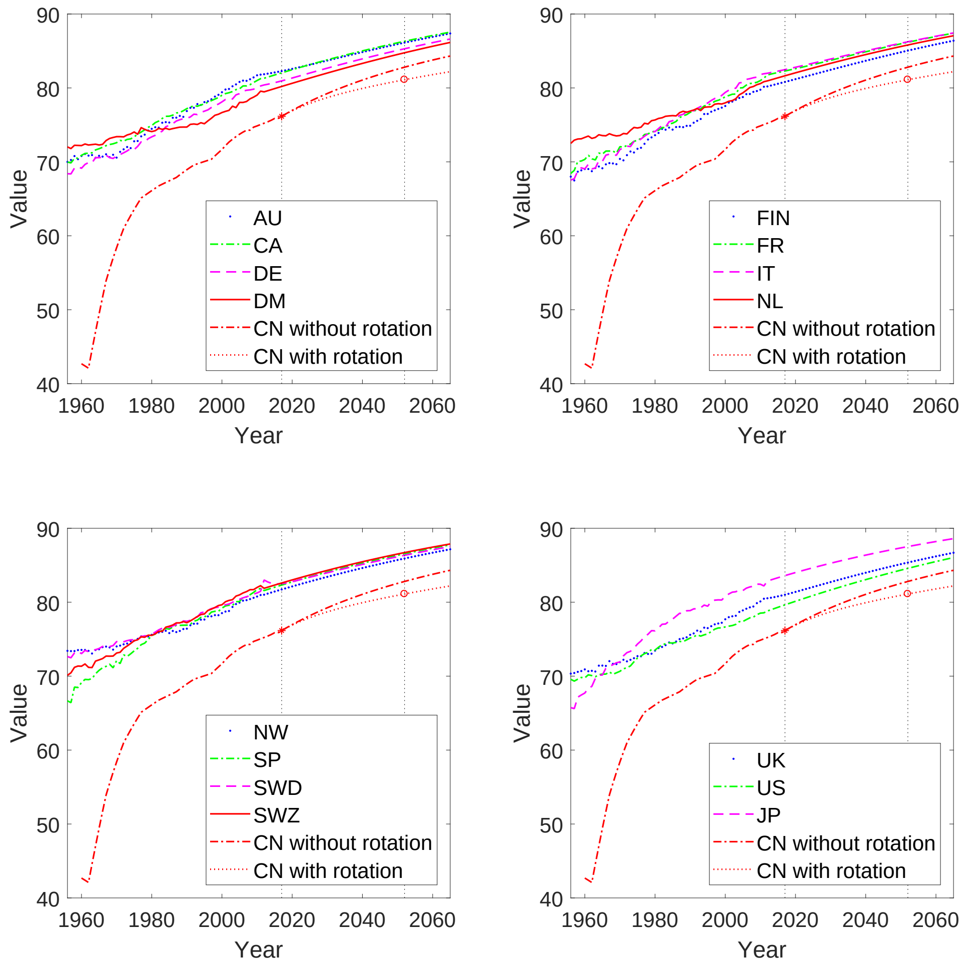


Figure 17: The estimated and projected (with/without rotation) e_0^{CN} obtained from the Lee-Carter model for China and the estimated and projected e_0 obtained from the Li-Lee model for 15 low mortality countries

5.3 Rotating the Age Effect

The rotation of the age effect follows directly from Li et al. (2013), and is similar to the rotation of the period effect. Again, to implement the rotation algorithm, we extend the

Lee-Carter model (for Chinese mortality) by allowing the age effect to vary with time:

$$\log m_{x,t}^{CN} = a_x^{CN} + b_{x,t}^{CN} k_t^{CN} + \varepsilon_{x,t}^{CN}. \quad (10)$$

The idea of rotating the age effect is to let $b_{x,t}^{CN}$ gradually rotate towards B_x, i.e., the systematic age effect of the 15 low mortality countries. Similar to the period effect, we let the rotation begin in year 2016, and let $b_{x,t}^{CN}$ reach B_x when $e_0^{CN}(t)$ hits the threshold e_0^u. Formally, for each $t \geq 2016$, $b_{x,t}^{CN}$ is given by

$$b_{x,t}^{CN} = \begin{cases} (1 - w_t) b_{x,0}^{CN} + w_t B_x, & \text{if } e_0^{CN}(t) < e_0^u, \\ B_x, & \text{if } e_0^{CN}(t) \geq e_0^u, \end{cases} \quad (11)$$

with $w(t)$ given in (9).

In this section, we focus exclusively on rotating the age effect, and keep the drift term of k_t^{CN} constant at d_o^{CN}. The weighting function has the same form given in Equation (8) - (9). First, we show the shapes of the weighting parameter in the Figure 18. The red line stands for the scenario where only the age effect rotates, while the blue line stand for the scenario where rotation occurs only for the period effect (which is the same line as in Figure 15). The red line indicates that $b_{x,t}^{CN}$ reaches B_x in 2029, which is faster than the case of the pure period effect rotation, which is not completed until around 2050. The speed of rotation depends on how fast $e_0^{CN}(t)$ increases towards the threshold e_0^u (see Equation 9).

The faster age effect rotation is due to two reasons. First, the period effect decreases faster in the absence of rotation (due to the the larger absolute value of d_o^{CN}). Second, the rotation will lead to higher age effects, and thus faster mortality improvements, for infants and children below 6 and elderly above 60, and lower age effects for teenagers and the working age population. Since the elderly have currently much higher mortality levels than the working age population, the rotation will lead to faster increase in projected life expectancy at birth. The faster increase in projected life expectancy at birth in turn leads to faster increase of the rotation weight $w(t)$, as can be seen in Equation (8) - (9).

Figure 19 displays the path of rotation for the age effect. We see that the rotation is completed in 2029, which is indeed earlier than 2048, the year of rotation completion when only the period effect is taken into account. The impact of the age effect rotation on the life expectancy at birth is shown in the Figure 20.

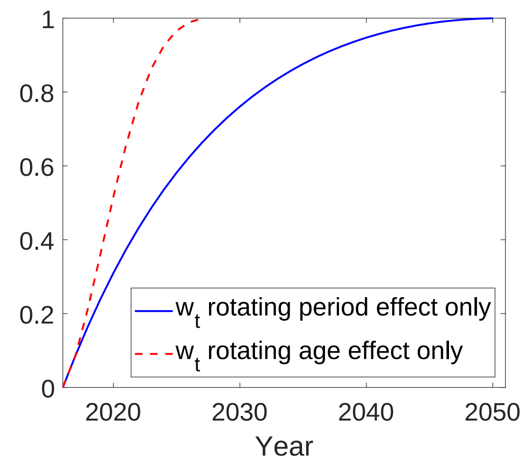


Figure 18: The comparison of the rotating $w(t)$ between (1) rotating the period effect (2) rotating the age effect.

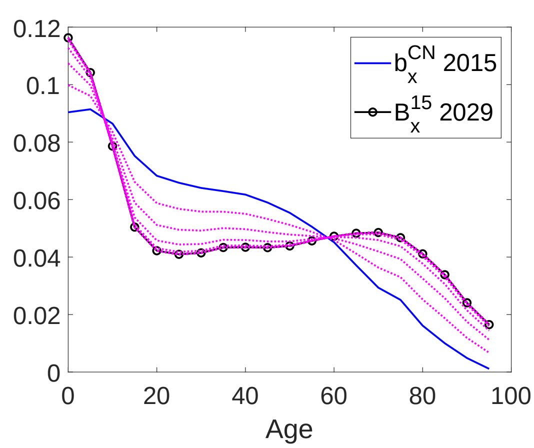


Figure 19: The original age effect of China in 2015, the rotating age effects of China between 2015 and 2029, and the ultimate age effect of benchmark mortalities.

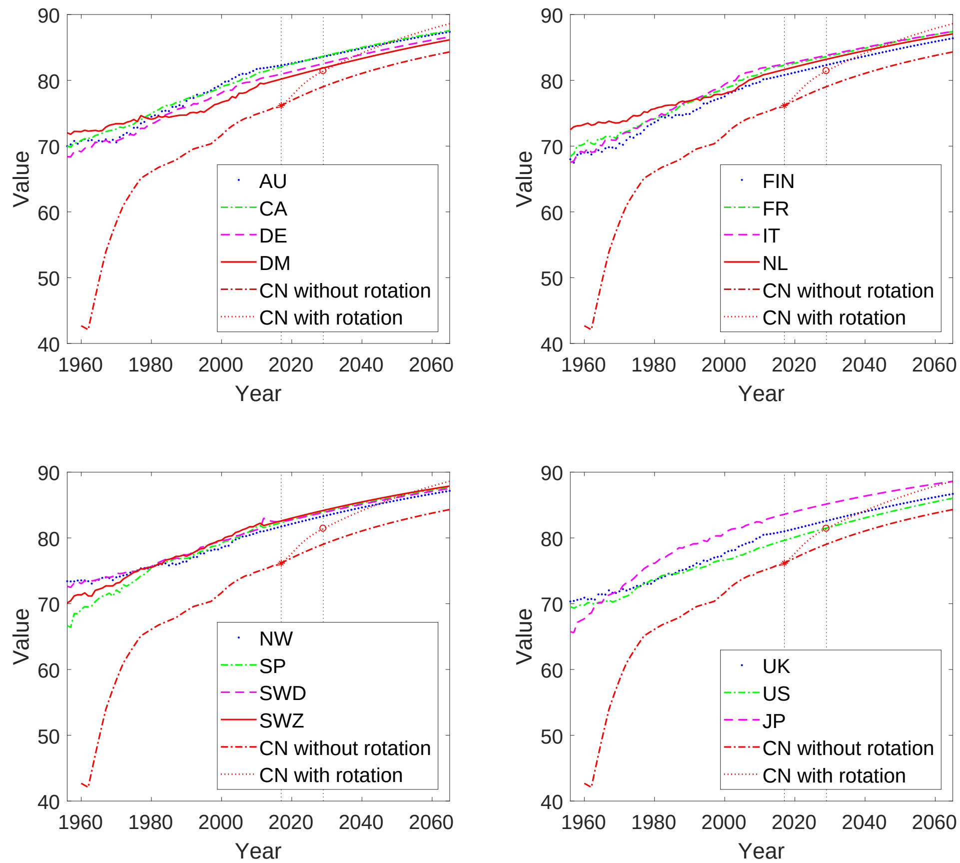


Figure 20: The estimated and projected (with/without age effect rotation) e_0^{CN} obtained from the Lee-Carter model for China and the estimated and projected e_0 obtained from the Li-Lee model for 15 low mortality countries.

5.4 Joint Rotation of Both Effects

In this section, we implement rotations on both the age effect the period effect. The shape of the weighting parameter, in comparison with the case of only age effect rotation and the case of only period effect rotation, is plotted in the Figure 21. Figure (22) displays the path of age effect rotation when only the age effect rotates (red dotted line) and when both the age and the period effect rotate (the green dotted) line.

The projected age-specific log mortality rates under different rotation algorithms are shown in Figure 24. We see that the Lee-Carter model without rotation leads to more substantial aggregate mortality decline, and more importantly, rather imbalanced mortality improvements across ages. In particular, while the mortality declines are huge at younger ages, they are projected to be very limited among the elders. In contrast, the projected $\log m_{x,2100}$'s using the Li-Lee model and the rotation algorithm are very similar, and are much more balanced across ages. Finally, the projected period life expectancy at birth under different rotation algorithms are shown in Figure 25.

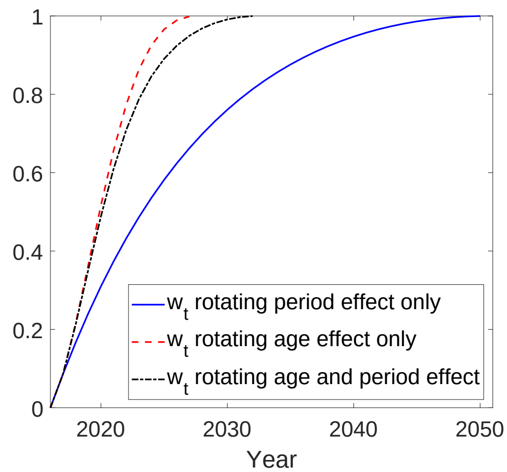


Figure 21: The comparison of the rotating $w(t)$ among (1) rotating the period effect, (2) rotating the age effect, and (3) rotating the period effect and the age effect.

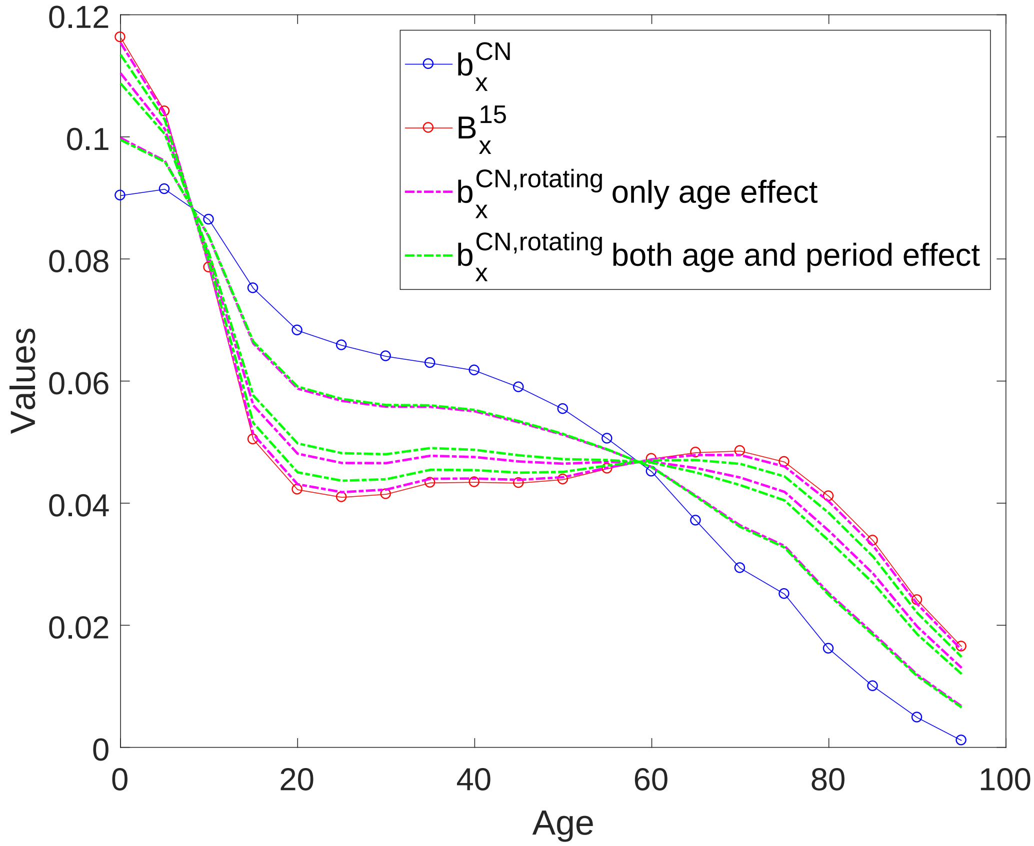


Figure 22: The original age effect of China in 2015, the rotating age effects(only age effect), the rotating age effects of China(both age effect and period), and the ultimate age effect of benchmark mortalities.

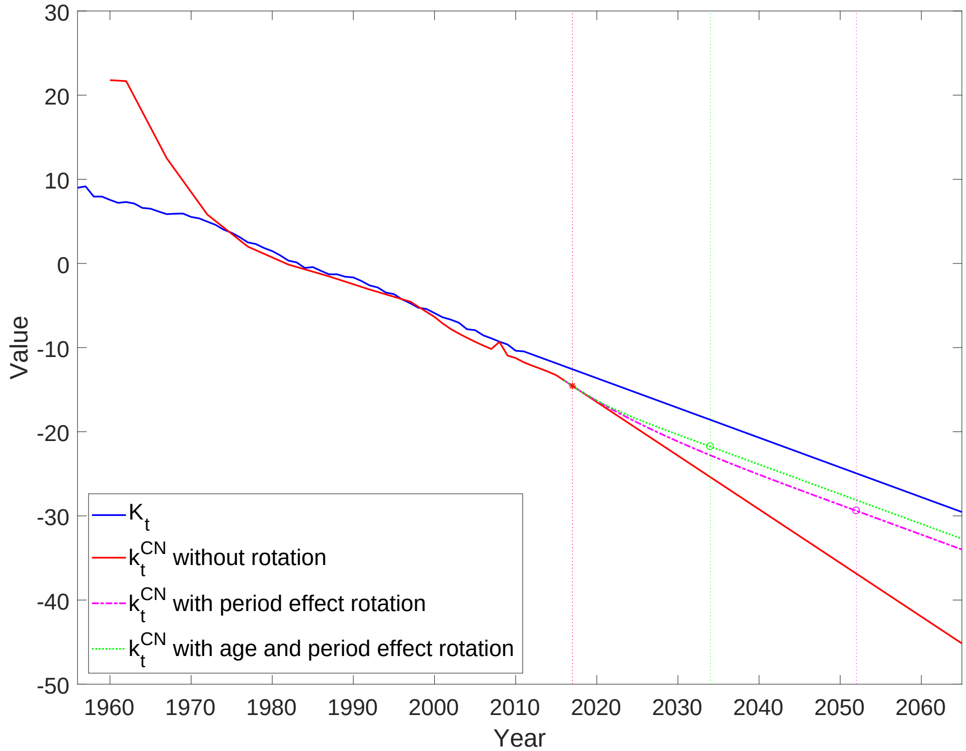


Figure 23: The estimated and projected (without rotation, with period effect rotation, and with age and period effect rotation) k_t^{CN} obtained from the Lee-Carter model for China and the estimated and projected K_t obtained from the Li-Lee model for 15 low mortality countries. Red dash line gives the first shared rotation point, green dash line shows the second rotation point for the rotation of both age and period effect, and the pink line takes us to the second rotation point for the rotation of only period effect.

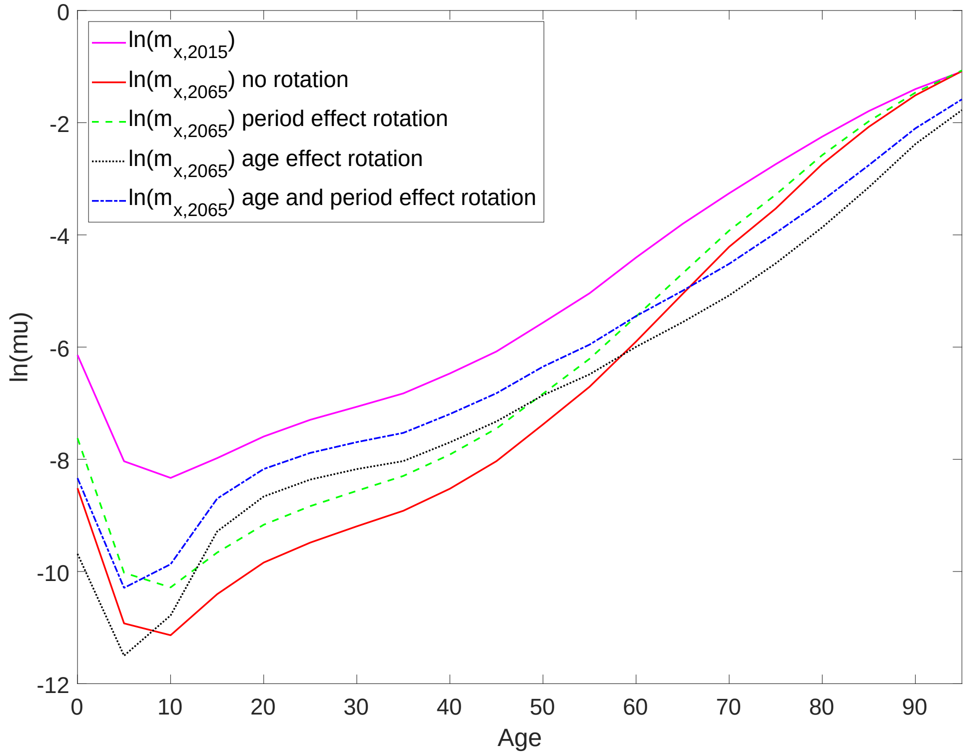


Figure 24: The comparison of the age-specific mortality rates for China among different rotation specifications.

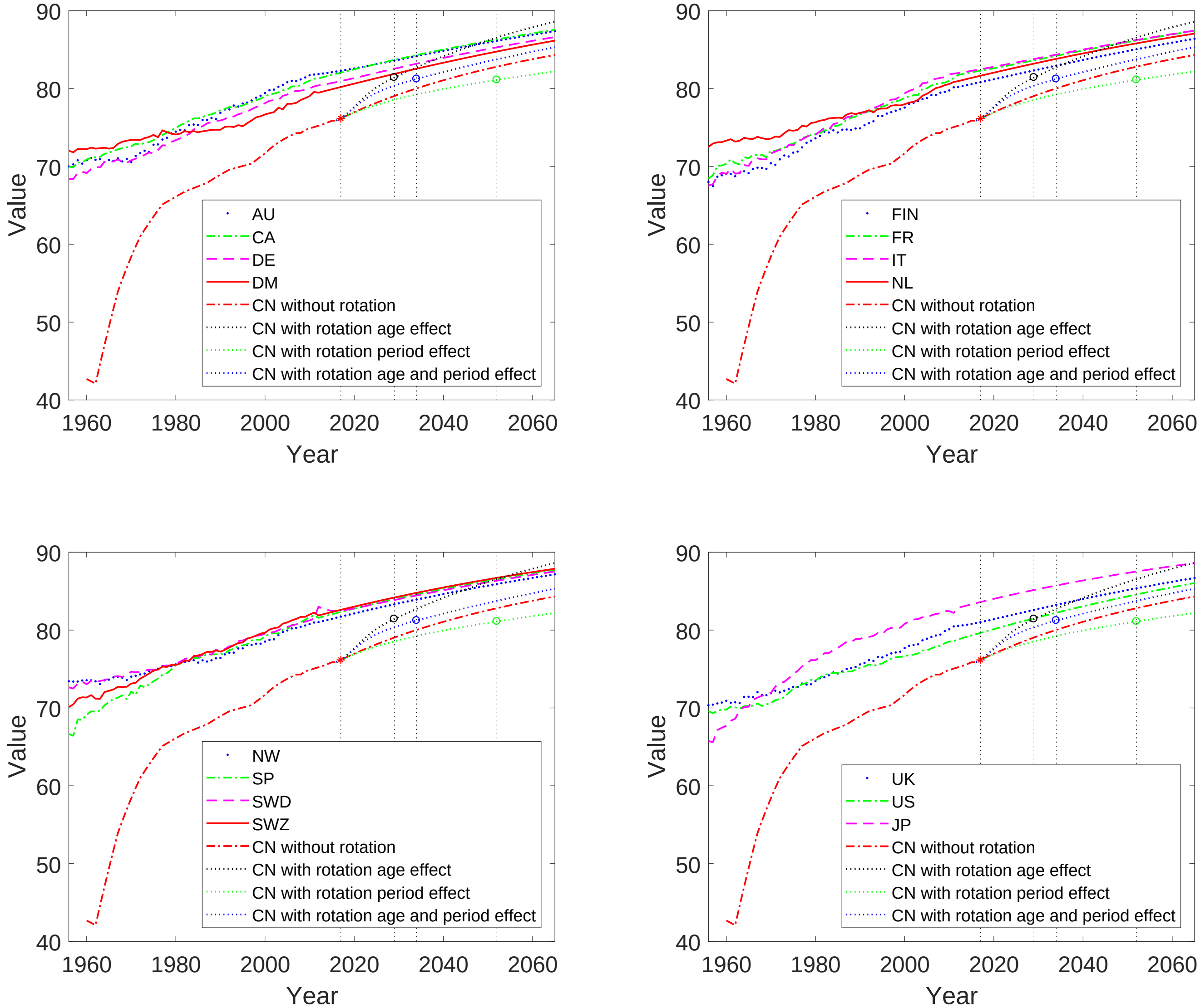


Figure 25: The estimated and projected (without rotation, with age effect rotation, with period effect rotation, and with age and period effect rotation) e_0^{CN} obtained from the Lee-Carter model for China and the estimated and projected e_0 obtained from the Li-Lee model for 15 low mortality countries.

6 Forecasting Fertility Rates in China

In this section, we show a natural follow-up to forecast fertility rates in China. Among the many existing methods, the Lee-Carter model (Lee and Tuljapurkar 1994) in fertility modeling is widely used by both practitioners and researchers. For example, Statistics Canada (Bohnert et al. 2015) applies the Lee-Carter model in projecting age-specific fertility rates in Canada; Myrskylä et al. (2013) extend the Lee-Carter model with "freeze-rate" extrapolation to project cohort fertility rate for various developed countries. In this study, we apply the Lee-Carter model for fertility rate projection.

Following Lee and Tuljapurkar 1994, we apply the Lee-Carter model with long-run mean constraint to project the age-specific fertility rates of China. In particular, The long-run mean of the period effect is selected as the long-run equilibrium level of total fertility rate, i.e., 2.1 births per woman. The choice of this ultimate mean of total fertility rate for China is supported by Alkema et al. (2011). The Lee-Carter model representation with long run mean constraint for age-specific fertility rates is given by:

$$F_{x,t} = a_x + b_x f_t, \tag{12}$$

where $F_{x,t}$ is the observed age-specific fertility rate for age x and year t; a_x is the baseline fertility level for age x, which is the average of $F_{x,t}$ over time; b_x characterizes the age pattern of fertility changes. In Equation (12), only f_t is assumed to evolve randomly over time. In other words, the uncertainty of age-specific fertility rate only manifest in its period effect of f_t. Equation (12) is estimated by the Single Value Decomposition (SVD) method. In particular, similar to the original Lee-Carter model, the following constraints are imposed:

$$\sum_x b_x = 1, \sum_t f_t = 0. \tag{13}$$

A representation of total fertility rate is then given by:

$$TF_t = \sum_x F_{x,t} = \sum_x a_x + f_t \tag{14}$$

The long-run mean constraint on total fertility rate is imposed by specifying a mean-reverting process with a designated mean for the period effect f_t. Following Lee and Tuljapurkar (1994), we use the mean-constrained autoregressive (AR) process to characterize the dynamics of f_tLee and Tuljapurkar (1994) [6]:

$$f_{t+1} = \mu + \rho f_{t-1} + u_t, \tag{15}$$

where μ carries the mean constraint for f_t, i.e., $\mu = (1-\rho)(TF_{ult} - \sum_x a_x)$, where TF_{ult} is set to 2.1 in our case.

Figure 26 shows the estimated a_x and b_x from China's fertility data. The baseline fertility rate a_x shows a hump-shaped fertility pattern, i.e., the golden ages of having a child lie between 20 and 30. Meanwhile, b_x's are the highest around age 25, meaning that the change of fertility pattern is the most profound for women around 25.

[6] Lee and Tuljapurkar (1994) use Autoregressive moving average process (ARMA) in their application. In this this study, we do not consider the moving average parts of their model because we primarily focus on the best estimates of the fertility rate.

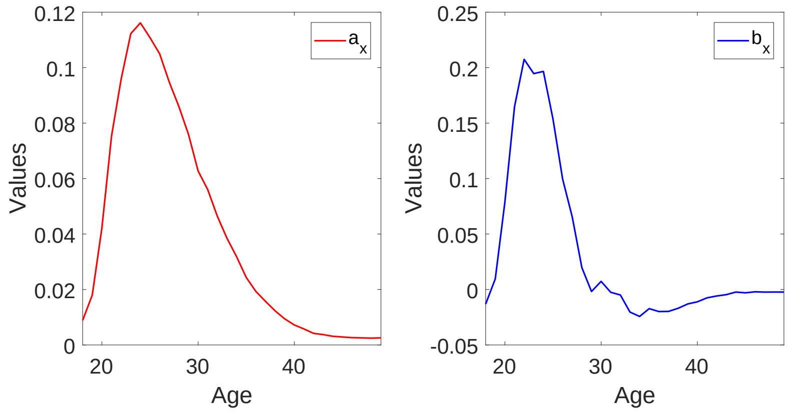


Figure 26: The estimated a_x and b_x obtained from the Lee-Carter model for the age-specific fertility rate of China.

Future age-specific fertility rates are then projected using Equation (15). The estimation and projections of f_t are presented in Figure 27. We see that the current value of f_t is lower than the pre-specified long-run mean, so the projected values of f_t will be increasing and converge to the long-run level over time.

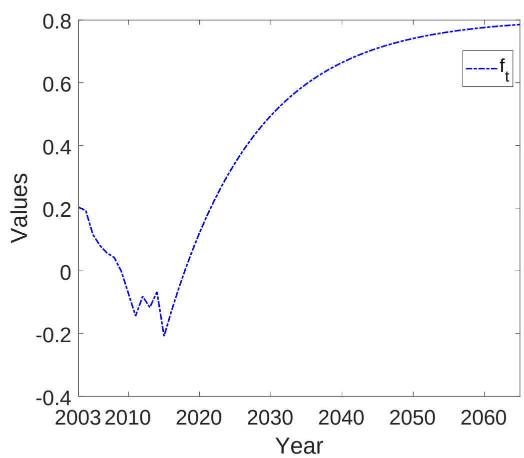


Figure 27: The estimated and projected f_t obtained from the Lee-Carter model for the age-specific fertility rate of China.

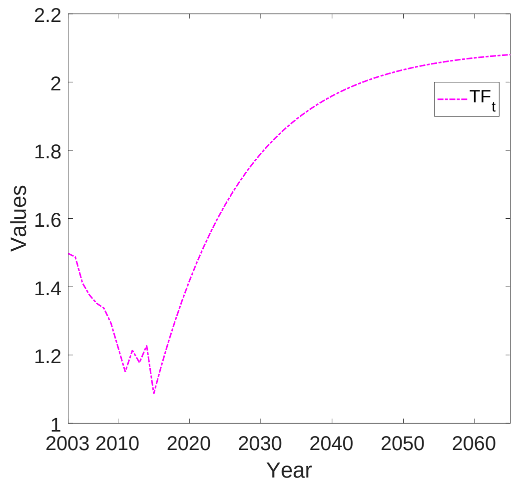


Figure 28: The estimated and projected TF_t obtained from the Lee-Carter model for the age-specific fertility rate of China.

Turning f_t into total fertility rate TF_t via Equation (14), we obtain the projections of the latter, which is shown in Figure 28. In this figure, we see that the total fertility rate will reach the long-run mean 2.1 at around 2060. With the recent abolish of the one-child policy, persistent decrease of total fertility rates, as observed in the past decade, is less likely to occur in the future.

7 Forecasting Population Structure in China

Now we proceed to the projection of population structure in China. In particular, we proceed with the following assumptions:

- We do not consider the impact of the ratio of males over females in the populations on the fertility pattern, i.e., the population structure is modeled as a one-sex birth-death process (Arnold et al., 2015).
- We focus on the deterministic population model.

- We assume the individual will decease at the age 100.
- We apply the unisex mortality to both genders.

Formally, the population structure could be summarized as the following McKendrick - Von Forester equations (see M'Kendrick (1925) and Foerster (1959)). Denote by $g^v_{x,t}, v \in \{Female, Male\}$ as the quantity of individuals with gender v and age x at time t, the mortality dynamics of the population structure is given by:

$$(\partial_x + \partial_t) \begin{bmatrix} g^m_{x,t} \\ g^f_{x,t} \end{bmatrix} = - \begin{bmatrix} \mu^m_{x,t} & 0 \\ 0 & \mu^f_{x,t} \end{bmatrix} \begin{bmatrix} g^m_{x,t} \\ g^f_{x,t} \end{bmatrix}, \tag{16}$$

i.e., the number of individuals decreases as the individuals age. We summarize the quantities for the two genders as $g_{x,t} = \begin{bmatrix} g^m_{x,t} \\ g^f_{x,t} \end{bmatrix}$. The fertility dynamics, on the other hand, is constructed by:

$$g_{0,t} = \Bigg(\int_{F_{agemin}}^{F_{agemax}} g^f_{x,t} f_{x,t} dx \Bigg) \begin{bmatrix} p \\ 1-p \end{bmatrix}. \tag{17}$$

In particular, $g_{0,t} = \begin{bmatrix} g^m_{0,t} \\ g^f_{0,t} \end{bmatrix}$ represents the number of male and female newborns at the time t, respectively. In our study, we set $F_{agemin} = 18$ and $F_{agemax} = 49$ as boundary conditions for the fertility dynamics. The choices of the boundary conditions for the fertility are discussed in Lee and Tuljapurkar (1994) and Myrskylä et al. (2013). p is the probability of being a male for a newly-born baby[7]. Obviously, given a starting point as $g_{x,0}$, the population structure evolves automatically with the combination of the mortality dynamics and fertility dynamics following Equation (16) - (17). We choose the starting point $g_{x,0}$ to be the China's male and female population in 2015 for each x.

A useful index that could be derived from the population structure is the old-age de-

[7] We set $p = 0.5353$, following the sex ratio at birth in 2016 from World Bank `https://data.worldbank.org/indicator/SP.POP.BRTH.MF?locations=CN`.

pendency ratio. The dependency ratio essentially characterizes the pressure of retirees on the working population. A lower dependency ratio indicates milder financial stress on the working population to sustain the retiring population, while a higher ratio indicates heavier financial stress. This ratio normally serves as an indicator for the government to monitor the sustainability of the social security system. In our setting, the dependency ratio is defined as the following[8]:

$$r_t = \frac{\sum_{x=65}^{99}(g_{x,t}^m + g_{x,t}^f)}{\sum_{x=15}^{65}(g_{x,t}^m + g_{x,t}^f)}, \tag{18}$$

where the retirement age is set to be 65. Figure 29 plots the dependency ratio of China over different projection horizons under all mortality rotation assumptions. More specifically, the red solid line stands for the benchmark case without any mortality rotation. The results from the benchmark case are comparable to the ones generated by the United Nations. The latter is represented by the solid line with circle markers. Moreover, the red dashed-dotted line, representing the case of period effect rotation, indicates that the dependency ratio will increase at the slowest rate and reaches 51%, which will be approximately 5% lower than the benchmark case. Meanwhile, the blue dashed line, i.e., projection under age effect rotation, predicts the fastest increase in the dependency ratio, which reaches 71 %, about 15 % higher than the benchmark case. Finally, the predicted dependency ratio under joint rotation reaches 62% in 2065, i.e., 12% higher than the benchmark. This joint rotation case is visualized by the blue dotted line.

In summary, the financial pressure of the old-age population is projected to be much more substantial under the assumptions that the mortality patterns of China would gradually rotate to the ones of the more developed countries. Moreover, existing mortality models, which potentially neglect the faster mortality improvements for the old-age population, are likely to underestimate the old-age dependence ratio of China.

[8]With focus of the population structure's impact on social security system, the dependency ratio in our setting only cover the population aged over 65. Some other authorities, e.g., World Bank, would also include the population age below 15 in the numerator of the dependency ratio

Another interesting indicator of population structure is the population pyramid. The population pyramid graphically portrays the distribution of age groups of a population. First, we generate the initial population pyramid based on the 2015 population structure of China in Figure 30[9], where the left panel represents the male population and the right panel represents the female population. Figure 30 shows that larger proportions of both male and female population fall between age 27 and 52. Correspondingly, the period when these population were born is 1963 to 1988. This period followed the *Great Chinese Famine* [10] and was ended by the tightening of the one-child policy.

The population pyramids of China under different mortality rotation assumptions in 2065 are plotted in the Figure 31. For ages between 0 and 50, the population is almost equally distributed. There is a huge downward jump at the age 50 because we model the fertility dynamics of China as a mean-reverting process that would return to its long-run mean from the historical low level in the early 21^{st} century. Moreover, different mortality rotation assumptions have much more noticeable impacts on the population structure above 75 than the younger population. In particular, rotating only the age effect (blue dashed line) inflates the population over 75 the most. The rank for these scenarios follows the same order as the old-age dependency ratio as shown in Figure 29. In this sense, the population pyramids yield consistent implications as the old-age dependency ratio.

[9] We use the population data from World Population Prospects: The 2017 Revision

[10] This famine could be visualized as a big sag between the age 50 and age 60

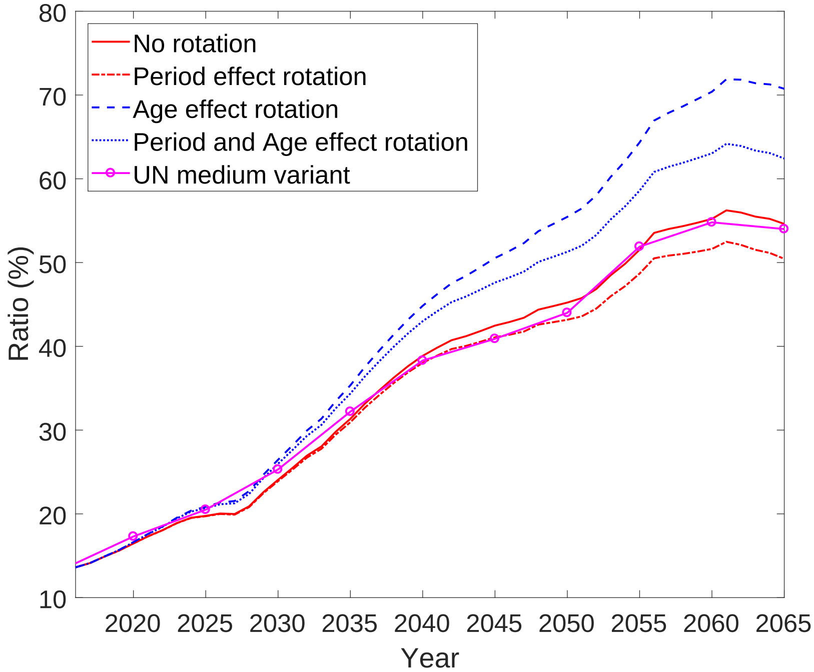


Figure 29: Dependency ratios under 4 scenarios: no rotation, period effect rotation, age effect rotation, and the joint rotation.

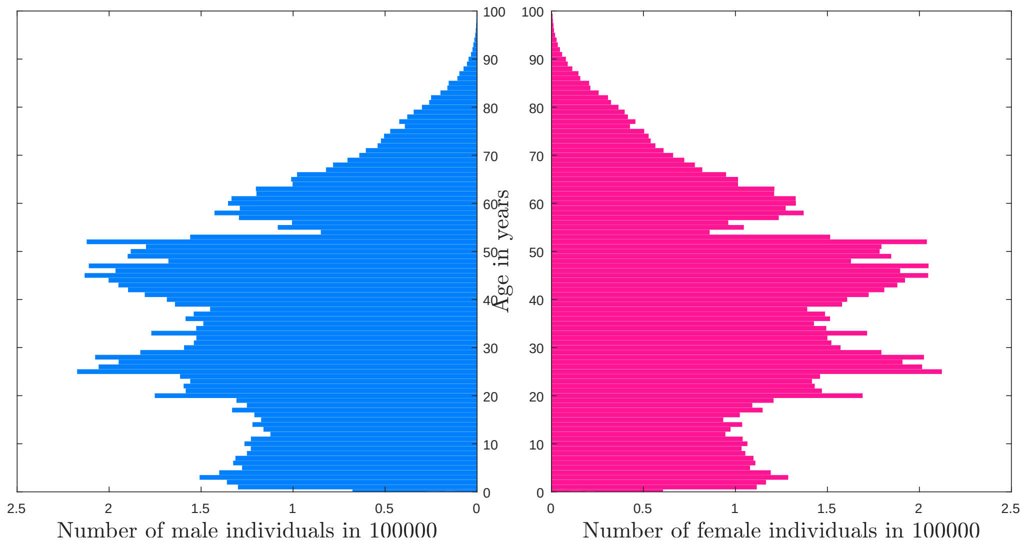


Figure 30: 2015 Population structure of China.

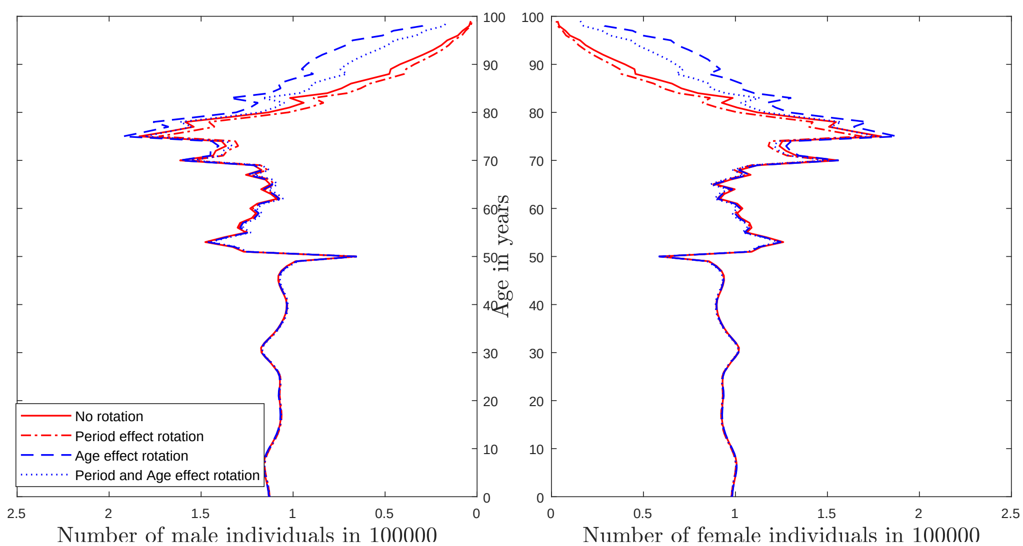


Figure 31: 2065 Population structure of China.

8 Implications on Social Security System

Now we study the implications of the mortality rotation assumptions on China's social security. The implications are illustrated with a simple and deterministic model of contributions and benefits. The details of this simplified social security system are gathered in Appendix B.

For a future year $T + \mu$, the contribution side of the social security system is given by:

$$R_{T+\mu} = r_{2016} * \left(\sum_{x=15}^{60} g_{x,T+\mu} \right) * cr_{2016} * 12, \tag{19}$$

where r_{2016} is the contribution per month per capita in year 2016, $g_{x,T+\mu}$ is the expected number of population in year $T + \mu$, and cr_{2016} is the implied coverage ratio of social security system in 2016. Moreover, the benefit side of the social security system is:

$$B_{T+\mu} = b_{2016} * \left(\sum_{x=61}^{x_{max}} g_{x,T+\mu} \right) * cr_{2016} * 12, \tag{20}$$

where x_{max} is set to be 99, and b_{2016} is the benefit per month per capita in year 2016. The gap of social security system in year $T + \mu$ is thus defined as:

$$G_{T+\mu} = R_{T+\mu} - B_{T+\mu}. \tag{21}$$

A negative gap means deficit in the social security system.

Based on China's population structure in 2016, we have $r_{2016} = 499.3$, $cr_{2016} = 0.771$, and $b_{2016} = 1116.8$. For derivations of these numbers we refer to Appendix B. Figure 32 shows the dynamics of social security gap under different rotation assumptions for China. Specifically, if we rotate the age effect and the period effect of China, the expected gap in the social security system will be negative 35 billion in 2065, i.e., 10 billion more deficit than the benchmark case.

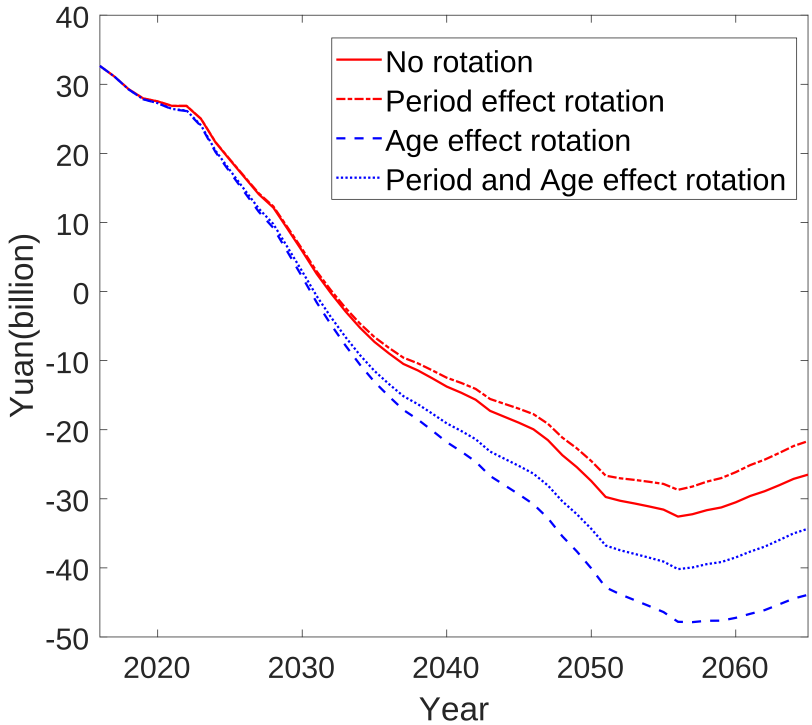


Figure 32: The gap in the social security system under 4 assumptions: no rotation, period effect rotation, age effect rotation, and age period effect rotation

9 Implications on Commercial Annuity Market

We see from the discussions above that the benchmark retirement income (111.68 CNY per month) cannot be met starting from 2030, solely based on social security contributions from the working population. Therefore, supplementary retirement products are in strong need to maintain a reasonable retirement income level for China's retirees in the following decades.

In this section, we illustrate the potential role that commercial annuity market could play in China's retirement benefit system under the following framework.

1. The target monthly retirement income is supposed to be 2373 yuan, which is the

monthly retirement income of the enterprise retirees of China in 2016[11].

2. Commercial annuity is the only source of retirement income besides the social security described in Section 8.

Let $e_{2016} = 2373$ be the targeted monthly retirement income for a retiree. ca_{2016} as the desirable amount of benefits received from the commercial annuity.

Given the second point of the setup for the commercial annuity market, we have $ca_{2016} + b_{2016} = e_{2016} = 2373$, where $b_{2016} = 1116.8$ and subsequently $ca_{2016} = 1256.2$ yuan. Following the above setting, the need for commercial annuity of each retiree, C_{T+1}, is given by:

$$C_{T+1} = ca_{2016} * \left(\sum_{x=61}^{x_{max}} g_{x,T+1} \right) * cacr_{2016} * 12, \tag{22}$$

where x_{max} is set to be 99 in our study, $cacr_{2016}$ is the implied coverage ratio of commercial annuity in 2016, ca_{2016} is the desirable amount fo benefits received from the commercial annuity in the year 2016.

Figure 33 displays the projected required amount of commercial annuity income to reach the targeted retirement income under the 4 mortality rotation assumptions. We see that the projected required amount of commercial annuity income will be increasing over the next 4 decades, and will reached the peak in around 2055. Moreover, if both the age effect and the period effect rotate, the required amount will be around 98 billion in 2065, which is approximately 8 billion higher than the benchmark projection (without rotation).

10 Conclusion

This report proposes a mortality rotation algorithm to project China's population structure. Implications of the mortality projection on China's social security system and the

[11] Annual report on China's social security development. Source: `http://www.mohrss.gov.cn/SYrlzyhshbzb/dongtaixinwen/buneiyaowen/201711/t20171124_282237.html`

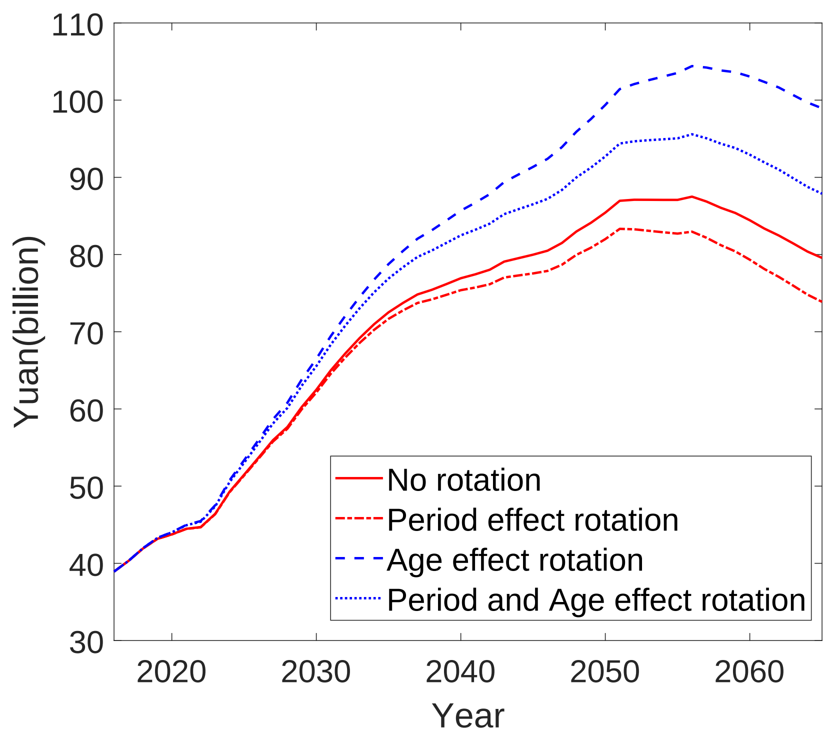


Figure 33: The total required amount of commercial annuity benefits under 4 mortality rotation assumptions: no rotation, period effect rotation, age effect rotation, and age period effect rotation.

commercial annuity sector are also discussed. While Chinese historical mortality patterns are generically different from the more developed countries, such discrepancies are likely to diminish along with future socioeconomic developments. Our approach incorporates the future changes of Chinese mortality patterns in the projection using a rotation algorithm. In particular, we allow the projected mortality patterns of China's population to be weighted averages of its own historical patterns and the benchmark patterns derived from a set of more developed countries. The weights of the benchmark values start from 0, and gradually increase to 1 as the projected life expectancy of China increases. Finally, coherence is achieved when the projected life expectancy reaches a threshold.

Moreover, China's age-specific fertility rates are projected, assuming that the total fertility rate would converge to the equilibrium level, 2.1, in the long-run. Based on the projected mortality and fertility rates, we find that a substantial proportion of China's population will lie between age 70 and 80 in 2065. Further, the social security system will be in deficit starting from 2030, with the deficit being the largest around 2055. In this case, substantial amounts of retirement benefit incomes from the commercial annuity sector are required to maintain a reasonable level of retirement income for the retirees.

Appendix

Kannisto-Thatcher Method for High Age Mortality

Kannisto-Thatcher (KT) method has been widely used to closing the mortality tables at high ages by many statistics authorities, e.g., Office for National Statistic (ONS) in UK and research institutes, e.g., the Max Planck Institute for Demographic Research, see Andreev et al. (2003). Under the assumption that the force of mortality m_x is a logistic function of the age x, KT method employs the logistic model to extend m_x to $m_{x_{max}}$ for the highest attained age x_{max} that is outside of the observed mortality table.

In our setting, we have the UN-WHO integrated mortality dataset of China for 5-age ranges (0-4, 5-9, ..., 75-79, 80+) and 1 year periods (1960, 1961, ..., 2015). We have a open age group 80+ in our data. We apply the KT method to generate $m_{x_{max}}$ that closes the open age group. We aim at matching our mortality dataset of China with the one of the group of more developed countries, especially for their 5-age ranges (0-4, 5-9, ..., 90-94, 95-99). Therefore, we select the age x_{max} to be 99.

Formally, the KT method to close the mortality table of our dataset at the age of 99 is as follows:

$$log(\frac{m_{x,t}}{1-m_{x,t}}) = \beta_{0,t} + \beta_{1,t}(x - x_0) + e_{x,t}, \tag{23}$$

where $x_0 = x^* - 20$. x^* is the lower bound of the open age group and equals to 80 in our case. x_0 turns to be 60. 20 is a typical choice in the KT method, see Wilmoth et al. (2007). Equation 23 is estimated over the age range that start from x_0 up to x^* for each year t. More specifically, 1-year age range (60, 62, ...,78, 79) is preferred for the estimation. However, our dataset features for 5-age range (60-64, 65-69, ..., 70-74, 75-79). In that sense, we leverage the Gompertz law of mortality, which is also broadly consider by many literatures e.g., Bongaarts (2005), to interpolate 5-age ranges into 1-age ranges (60, 61, ..., 78, 79) for each year t. Detailed derivations and discussions are referred to Olshansky and Carnes (1997) and Bongaarts (2005).

The estimation of Equation 23 gives $\hat{\beta}_{0,t}$, $\hat{\beta}_{1,t}$ for each year t, with which we can extend the mortality table from up to 99, i.e., $\hat{m}_{80,t}, \hat{m}_{81,t}, ..., \hat{m}_{98,t}, \hat{m}_{99,t}$ are at hand. The hat of the central mortality rate represents the predicted values given by the KT method, which can be formulated as follows:

$$log(\frac{\hat{m}_{\tilde{x},t}}{1-\hat{m}_{\tilde{x},t}}) = \hat{\beta}_{0,t} + \hat{\beta}_{1,t}(\tilde{x} - x_0). \tag{24}$$

Where $\tilde{x} \in \{80, 81, ..., 98, 99\}$ and $x_0 = 60$.

Last but not least, we tune these extended mortality rates for 1-age range (80, 81, ..., 98, 99) to the ones for 5-age range (80-84, 85-89, 90-94, 95-99), which is in line with the observed ones for 5-age range (0-4, 5-9, ..., 70-74, 75-79). It is conducted in the following way :

- We transform the central mortality rate $\hat{m}_{80,t}$[12] to the death probability $\hat{q}_{80,t}$ by $\hat{q}_{80,t} = 1 - exp(-\hat{m}_{80,t})$ and we repeat this procedure for other ages, see Pitacco et al. (2009)

- $\hat{q}_{80-84,t}$ is obtain via $1 - \left(1 - \hat{q}_{80-84,t}\right)^5 = 1 - \left(\prod_{s=0}^{4}(1 - \hat{q}_{80+s,t})\right)$ under the uniform assumption of the force of mortality during the 5-age interval ((Wilmoth et al., 2007)).

- $\hat{m}_{80-84,t}$ is given by $exp(-5 * \hat{m}_{80-84,t}) = (1 - \hat{q}_{80-84,t})^5$, following the same intuition of the first bullet point. Similarly, we get $\hat{m}_{85-89,t}$, $\hat{m}_{90-94,t}$, $\hat{m}_{95-99,t}$.

In total, we have $m_{0-4,t}$, $m_{5-9,t}$,..., $m_{75-79,t}$, $\hat{m}_{80-84,t}$, $\hat{m}_{85-89,t}$, $\hat{m}_{90-94,t}$, $\hat{m}_{95-99,t}$, for $t \in \{1960, 1961, ..., 2014, 2015\}$.

[12] To simply the notation, we use mortality rates ($\hat{m}_{80,t}$,$\hat{m}_{81,t}$, ...,$\hat{m}_{84,t}$) as an example.

The Social Security System Specification

First, we derive the key components of the social security system from the official report given by the Ministry of Human Resources and Social Security[13]. First we define the variables used in the equations. Let n_t^p represent total number of individuals participate in the social system, including the contributors and beneficiaries. From the report, we known $n_{2016}^p = 887.77$ million. Let $N_{x,t}$ be the number of population at the age x in the year t. From the growth rate obtained from the world bank and the 2015 number from UN, we know $N_{15+,2016} = 1151.20$ million, where 15+ represent the individuals age over 15 year old with assumption that the individual start to participate social security system at the minimum age of 15[14]. The implied coverage ratio of social security system would be $cr_t = \frac{n_t^p}{N_{15+,t}}$. In our case, we fix the coverage ratio at the year 2016 in the projections of our social security system, i.e., $cr_{2016} = \frac{n_{2016}^p}{N_{15+,2016}} = \frac{887.77}{1151.20} = 0.771$.

For the benefit side of the social security system, let n_t^b represent the numbers of beneficiaries and $n_{2016}^b = 253.73$ in 2016. Besides, B_t is total benefit paid by the social security system at year t and $B_{2016} = 3400400$ million. b_t is the benefit per month per capita in year t, which in 2016 is calculated as $b_{2016} = \frac{B_{2016}}{12 * n_{2016}^b} = \frac{3400400}{12*253.73} = 1116.8$ yuan.

Naturally, for the contribution side the number of contributor comes as $n_t^c = n_t^p - n_t^b$. Then, in 2016, $n_{2016}^c = n_{2016}^p - n_{2016}^b = 887.77 - 253.73 = 634.04$ million. In addition, R_t is total benefit paid by the social security system at year t and $R_{2016} = 3799100$ million. r_t is the benefit per month per capita in year t, which in 2016 is calculated as $r_{2016} = \frac{r_{2016}}{12 * n_{2016}^c} = \frac{3799100}{12*634.04} = 499.3$ yuan.

Details of the social security system used in this report is presented below.

- What is the proportion of the total number of individuals aged between 15 and 60 that contributes to the social security system? On one hand, from the report, we know the

[13] http://cj.sina.com.cn/article/detail/6018289492/264033

[14] Actually, it should be 18. We make a compromise here because we observe the population data in a 5 year range, i.e., we only have the data on 15-19 age group.

number of the individuals that contribute to the social security is $887.77 million - 253.73 million = 634.04 million$, in 2016. On the other hand, we update the number of individuals aged between 15 and 60 from United Nations in 2015, i.e., 935.2681 million, to the one year 2016 by assuming the same growth rate for 15-60 as the total population, i.e., $935.2681 * (1 + 0.58\%) = 940.7255$ million[15] . We could not use our data from the statistics yearbooks because it is only 1% survey sample. From the number above, we know the cover rate of social security in age range 15 - 60 of total population, i.e., $\frac{634.04}{940.7255} = 67.4\%$. We keep this cover rate fixed in the future projection of population structure.

- What is the proportion of the total number of individual aged above 60 that receive benefits from social security system ? From the report, we know that there are 253.73 million individuals who receive benefits from the social security system. Follow the similar logic in the first bullet point, we obtain the total number of individuals that age over 60 in the whole population from the UN unisex population structure withe growth rate obtained from the World Bank, i.e., $214.739 * (1 + 0.58\%) = 215.992$ million. It follows naturally the interested ratio. We can't calculate, simply because we have more individuals who take the benefits than total number.

- Although we can not calculate the contribute ratio and the benefit ratio separately, we can use a uniform coverage ratio, $88777/115120.9 = 0.771$, where the denominator is the population of China at 2016 over 15 years old, using the growth rate above.

- In addition, we also need to calculate the contribution and benefit that each participants would need to pay and receive. We begin with calculating contribution, i.e., $\frac{37991*100}{(887.77-253.73)*12} = 499.32$ yuan. As we know the total expenditure of social security system is 34004*100 million yuan. The benefit is set to be $\frac{34004*100}{253.73*12} = 1168.8$ yuan.

[15] The growth rate is obtained by from the World Bank, as $1.0058 = \frac{1.379}{1.371}$, see https://data.worldbank.org/indicator/SP.POP.TOTL?locations=CN

- In that sense, the dynamic of this social security system goes as follows. For the year t, social security system receives its income from the contribution side, i.e., $+N_{15-60,t} * 77.1\% * 499.32 * 12$, and makes its payment to the benefit side, $-N_{60+,t} * 77.1\% * 1168.8 * 12$, where $N_{x,t}$ represent the number of individuals at the age x and year t.

- The gap in the social system is then
  $N_{15-60,t} * 77.1\% * 499.32 * 12 - N_{60+,t} * 77.1\% * 1168.8 * 12$.

References

Leontine Alkema, Adrian E Raftery, Patrick Gerland, Samuel J Clark, François Pelletier, Thomas Buettner, and Gerhard K Heilig. Probabilistic projections of the total fertility rate for all countries. *Demography*, 48(3):815–839, 2011.

K Andreev, D Jdanov, E Soroko, and V Shkolnikov. Methodology: Kannistoˆ thatcher database on old age mortality. *Max Planck Institute for Demographic Research, Rostock, Germany.[Online]*, 2003.

Séverine Arnold, Alexandre Boumezoued, Héloïse Labit Hardy, and Nicole El Karoui. Cause-of-death mortality: What can be learned from population dynamics? 2015.

Nora Bohnert, Jonathan Chagnon, and P Dion. *Population projections for Canada (2013 to 2063), provinces and territories (2013 to 2038): technical report on methodology and assumptions*. Statistics Canada= Statistique Canada, 2015.

John Bongaarts. Long-range trends in adult mortality: Models and projection methods. *Demography*, 42(1):23–49, 2005.

Heather Booth and Leonie Tickle. Mortality Modelling and Forecasting: A Review of Methods. *Annals of Actuarial Science*, 3(1-2):3–43, 2008.

Heather Booth, John Maindonald, and Len Smith. Applying lee-carter under conditions of variable mortality decline. *Population studies*, 56(3):325–336, 2002.

Andrew JG Cairns, David Blake, and Kevin Dowd. A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration. *Journal of Risk and Insurance*, 73(4):687–718, 2006.

von J Foerster. The kinetics of cell proliferation. *New York: Grune and Stratton*, 1959.

Rob J Hyndman, Heather Booth, and Farah Yasmeen. Coherent mortality forecasting: the product-ratio method with functional time series models. *Demography*, 50(1):261–283, 2013.

Väinö Kannisto. The advancing frontier of survival: life tables for old age. 1996.

Ronald D Lee and Lawrence R Carter. Modeling and forecasting us mortality. *Journal of the American statistical association*, 87(419):659–671, 1992.

Ronald D Lee and Shripad Tuljapurkar. Stochastic population forecasts for the united states: Beyond high, medium, and low. *Journal of the American Statistical Association*, 89(428):1175–1189, 1994.

Hong Li and Johnny Siu-Hang Li. Optimizing the lee-carter approach in the presence of structural changes in time and age patterns of mortality improvements. *Demography*, 54 (3):1073–1095, 2017.

Nan Li and Ronald D Lee. Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. *Demography*, 42(3):575–594, 2005.

Nan Li, Ronald Lee, and Patrick Gerland. Extending the lee-carter method to model the rotation of age patterns of mortality decline for long-term projections. *Demography*, 50 (6):2037–2051, 2013.

AG M'Kendrick. Applications of mathematics to medical problems. *Proceedings of the Edinburgh Mathematical Society*, 44:98–130, 1925.

Mikko Myrskylä, Joshua R Goldstein, and Yen-hsin Alice Cheng. New cohort fertility forecasts for the developed world: rises, falls, and reversals. *Population and Development Review*, 39(1):31–56, 2013.

S Jay Olshansky and Bruce A Carnes. Ever since gompertz. *Demography*, 34(1):1–15, 1997.

Ermanno Pitacco, Michel Denuit, and Steven Haberman. *Modelling longevity dynamics for pensions and annuity business*. Oxford University Press, 2009.

Adrian E Raftery, Jennifer L Chunn, Patrick Gerland, and Hana Ševčíková. Bayesian probabilistic projections of life expectancy for all countries. *Demography*, 50(3):777–801, 2013.

A Roger Thatcher, Väinö Kannisto, and Kirill F Andreev. The survivor ratio method for estimating numbers at high ages. *Demographic Research*, 6:1–18, 2002.

John R Wilmoth. Computational methods for fitting and extrapolating the lee-carter model of mortality change. Technical report, Technical report, Department of Demography, University of California, Berkeley, 1993.

John R Wilmoth, Kirill Andreev, Dmitri Jdanov, Dana A Glei, C Boe, M Bubenheim, D Philipov, V Shkolnikov, and P Vachon. Methods protocol for the human mortality database. *University of California, Berkeley, and Max Planck Institute for Demographic Research, Rostock. URL: http://mortality. org [version 31/05/2007]*, 9:10–11, 2007.

World Health Organization. The world health report: 1999: Making a difference: message from the director-general. 1999.

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